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Multi-Fidelity Design Optimization Studies for Supersonic Jets Using Surrogate Management Frame Method

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Design of the realistic supersonic jets has been challenging problem. The design space for a ground boom is known to be non-smooth and discontinuous. Due to those particular characteristics as such, gradient-based optimizer can not be employed to locate the global minimum, and gradient-free search method is more suitable for an extensive exploration in design sites. In this study we employ one of the direct search method, pattern search method which has been used for a general constrained optimization on filter methods. A general pattern search (GPS) method[38] is the search algorithm which composes of the *search step* and the local *poll step*. Random global search is performed in the search step. In the poll step, the search directions are selected in a way that they can accelerate the convergence of search. A filter algorithm is combined with the PS to handle the mission constraints in a aggregate way. Moreover, to overcome the restriction to a finite set of directions in the GPS for local exploration, mesh adaptive direct search (MADS) method is introduced providing more search directions. A virtual mesh on the design space is constructed and as the algorithm iterates, the mesh can be refined or coarsened corresponding to the success of the search.

This method is applied to the shape optimization of the low boom supersonic jet and the pattern search proceeds through the several iterations, until all the mission constrains are satisfied and aerodynamic/boom performance is improved. The objective function is the minimization of MTOW and the low ground boom is treated as one of the constraints.

I. Introduction

When the design space behaves very unexpectedly to make it impossible to get a sensitivity information over the whole domain, the direct search methods are suitable as they do not compute or approximate derivatives. Direct search method are a class of global search method ,and robust and simple to implement. However major drawback is the prohibitively expensive computational cost for a large number of function evaluation. Therefore it is difficult to proceed global optimization enough to explore the domain thoroughly. On the other hand, local search method has an advantage over the global search method with the fast convergence as it is employing derivative information for a search direction. For a space which maintains multiple local minima however if local smoothness can be guaranteed, the combination of global and local search will be ideal in a way that exploratory search over the domain can be performed by global step and then gradient-based local search can be applied to a identified sites. Previous work showed the two-level optimization,[37] which incorporated adjoint-method-based local optimization following the first level of global optimizations. One of the advantages of this two-level optimization allows a larger number of design variables in the second level optimization than in the first level.

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More function evaluation can refine the identified design site to accelerate the overall convergence. In this study, we employ one of the surrogate-based direct search methods, called mesh adaptive direct search method(MADS),[39] which is another form of pattern search method(GPS) for general constrained optimization.[38] Design space is composed of the meshes, and the objective functions and constraints are evaluated on the mesh points. This is an iterative search algorithm where surrogate model updated in each iteration by evaluating trial points, ensuring the convergence to an optimum. This algorithm is suitable for the nonlinear optimization especially for the noisy objective function, as this does not compute or approximate any derivatives, penalty constriants, or Lagrange multipliers. In order to handle the constraints, it considers the sum of weighted constraint violations as another objective function, and apply the concept of non-dominance, called *filter* in this algorithm, in a multi-disciplinary optimization aspect. The algorithm is divided into SEARCH STEP and POLL STEP. In a SEARCH STEP, surrogate-based direct search is applied and it explores the design space globally. If the mesh found in the SEARCH STEP is dominated by any of the mesh points, then a POLL STEP is invoked and neighboring points around the current incumbent are constructed in the directions of positive basis. The incumbent point is called a poll center. The poll center and neighboring points are called a *pattern* or a *frame*. In each iteration, a feasible point set and a infeasible non-dominated point set are kept and updated, and the mesh point found by SEARCH STEP or POLL STEP is rejected or accepted by the filter condition. Corresponding to the success of the SEARCH STEP or POLL STEP, a mesh size and the poll size are updated. We can terminate the iteration either when an improved feasible point is found.

The use of the accurate surrogate model in the global search is critical because the accurate representation of the aerodynamic/boom design space requires expensive high-fidelity analysis, and the direct combination of the high-fidelity analysis with the global search in the SEARCH STEP is prohibitively expensive. We construct the multi-fidelity response surface which consists of a large number of low-fidelity analysis and a moderate number of high-fidelity analysis. High-fidelity analysis applies only where low-fidelity analysis can not resolve phenomena accurately. PASS has the simple module of aerodynamic analysis which is based on the assumptions for classical aerodynamics. This form of the PASS provides low-fidelity analysis. However it has all separate modules for mission analysis such as propulsion, stability, weight estimation and etc. The PASS is connected with our optimizer, the GA, by providing non-linear constraints form for mission analysis. Our starting baseline configuration is generated by this low-fidelity level of the PASS. However the mission analysis based on the classical aerodynamics can not be trusted, as the high-fidelity can provide the more accurate aerodynamic analysis. The PASS combined with the high-fidelity analysis tool is our real model as opposed to the surrogate model. The true optimum value should be feasible by the high-fidelity analysis. Since GA requires a large number of function evaluations, mission analysis of the PASS with the direct connection of the high-fidelity tool will be very expensive. Thus a surrogate model for the high-fidelity is introduced in our study, and our variable-fidelity analysis tools build multi-fidelity response surface for drag estimation and ground boom prediction. For the mesh point evaluation, high-fidelity analysis is applied, and response surface is making a role to give a inexpensive approximation to the real function analysis. We use a Kriging response surface for its capability to handle the rather noisy design space and to capture the possible multiple local minima.

The GA has been well-known global optimizer. It is a evolutionary algorithm based on the genetic operations such as crossover, mutation and reproduction. It is robust and independent of the problem but slow in the convergence resulting from a large number of necessary function evaluation. Combined with inexpensive response surface, it can be employed as a global search method. The GA used in this study has a capability of handling multi-objective optimization problem with several constraints. Ground boom noise level is included in one of the constraint in this study rather than having it as another objective function, however we can have it as another objective function along with MTOW. Then our optmization problem becomes multi-objective problem and previous work [20] handled this type of problem successfully.

In our study, the minimization of MTOW while preserving good aerodynamic performance and satisfying all the mission requirements including low ground boom, are sought.

II. DESIGN METHOD : Surrogate Management Frame

A. General Pattern Search Filter Method

Pattern search algorithms are a class of direct search method. For a non-smooth design space for such as ground boom, a direct search with global scope is ideal as they do not need derivative information.

This is an iterative search method, and each iteration includes *patterns* characterized by the direction of the step and a step length parameter, where the step is determined by the two points on the mesh. The mesh on the design space is virtual and true function values are evaluated only for the points on the mesh. The mesh can be refined or coarsened as iteration proceed corresponding to the success of finding improved mesh points.

Each iteration consists of two stages. The first, SEARCH STEP makes a global exploration of the design space, and a large number of function evaluations are required for a global search which make computational cost prohibitively expensive. Therefore a surrogate-based search algorithm is crucial in the application of this type of direct search method. Multi-fidelity response surface is employed in this study and the details of building a response surface will be explained in the later section.
The second, POLL STEP makes a local exploration near an incumbent points identified by the search step and enable the theory to guarantee convergence.[40] Neighboring points are polled in the directions where convergence can be guaranteed, and these poll set and the incumbent x_k are called *frame*. The direction of polling is a positive spanning set of $\mathbf{R^n}$. If the gradient of vector at current x_k is not zero, then at least one vector in the positive basis defines a descent direction for f from x_k.[41,40]Detailed convergence analysis is based on Clarke's calculus[42] for non-smooth functions. By examining the descent direction around the x_k, we can check if the current x_k is a local minimum on the mesh or can provide the descent direction to accelerate the convergence. Positive basis has at least $n+1$ and at most $2n$ vectors and they are called minimal positive basis and maximal positive basis, respectively. In this study, we use maximal positive basis which encompass all coordinate directions, both positive and negative.

Mesh can be defined in a mathematical way, $M(x_k, \Delta_k) = \{x_k + \Delta_k Dz : z \in \mathbf{N^{n_D}}\}$, where Δ_k is the mesh size parameter, and n_D is the number of columns of the matrix D. Matrix D is a finite matrix whose columns in $\mathbf{R^n}$ from a positive spanning set.

Once poll frame is constructed, the function values are evaluated at the neighboring mesh points around x_k. If the poll step fails in finding an improved mesh point on the mesh, the mesh is then refined and x_{k+1} is set to x_k. If either the SEARCH or POLL step is successful in locating an improved mesh point $x_{k+1} \neq x_k$, then mesh size parameter is kept the same or increased, and the iteration can proceed until the satisfactory reduction is produced in objective function.

However in the problem of constrained optimization such as in our application, strict reduction in the objective function is meaningless if constraint violation makes the design infeasible. Therefore filter algorithm[38] is introduced in this study and the optimization problem resembles multi-disciplinary aspects by treating the aggregate violation of the constraints as another form of objective function. The notion *dominance* is similar to the concept of a *filter* in this algorithm. For a pair of vectors x, x', x *dominates or filters* x', if and only if for all i, $x_i \leq x'_i$, and $x \neq x'$. We apply filter condition to constraint violation, $h(x)$ and objective function, f for the mesh points. A constraint violation function is defined, $h(x) = \|C(x)_+\|$, where $h(x) = 0$ if and only if $C(x) \leq 0$, and $h(x) > 0$if and only if $C(x) \not\leq 0$.

The criteria for the successful step is determined by the filter/non-dominance. In either search step or poll step, if unfiltered point was found, we can consider the step successful, and update the parameters and the mesh correspondingly. The points of interest in the constrained design space consist of the best feasible point found so far and the least infeasible point with the best function value found so far. Either one of two sets can contain the poll center at the poll step.

A filter $\mathcal{F}$ is a finite set of unfiltered infeasible points in $\mathbf{R^n}$, and $\bar{\mathcal{F}}$ is a set of filtered points. Let f_k^F be the best function value found so far at an infeasible point (point 1 in the Figure 1, and f_k^I be the smallest objective function value of the points (point 2 in the Figure 1) whose constraint violation function values are equal to h_k^I which is the smallest constraint violation in the unfiltered set. Figure 1 shows two incumbent points in each iteration. In each iteration, the mesh point found by surrogate-based optimizer is evaluated. If it is filtered by the point in the $\mathcal{F}$, then it is rejected and if it is unfiltered, then it is included in $\mathcal{F}$.

The summary of filtered pattern search method is following.

- INITIALIZATION Let $\mathcal{F}_0$ be the filter associated with a set of initial points on the mesh. x_0 is the initial incumbent point in the $\mathcal{F}_0$ and Δ_0 is the initial mesh size parameter.

- SEARCH and POLL STEP Perform the SEARCH and possibly POLL steps until an unfiltered trial point x_{k+1} is found, or until it is shown that all trial points are filtered by $\mathcal{F}_k$.

  - SEARCH STEP: Evaluate $h(x)$ and f on a set of trial points on the current mesh.
  - POLL STEP: Evaluate $h(x)$ and f on the POLL set current incumbent point.

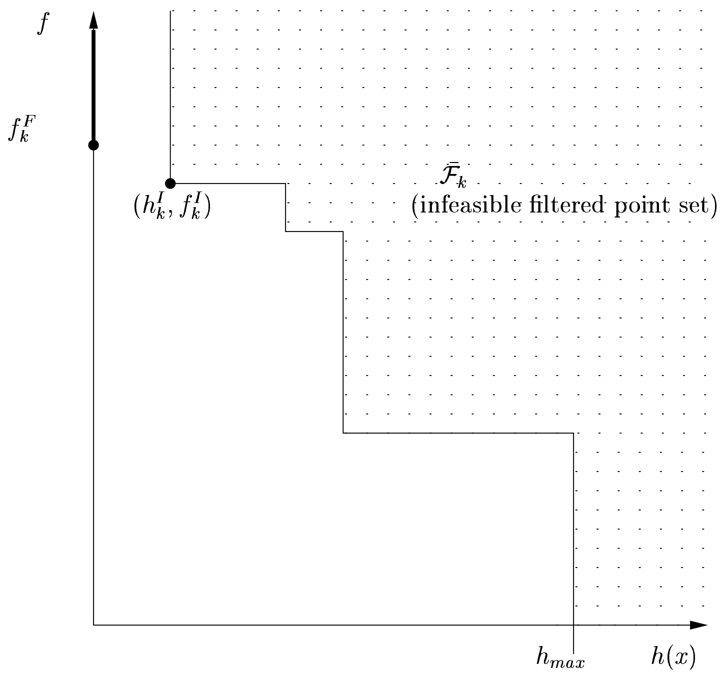


Figure 1. Feasible incumbent point f_k^F, infeasible incumbent pointf_k^I, and infeasible filtered points $\bar{\mathcal{F}}_k$, taken from the paper [38]

- PARAMETER UPDATE Include the all the trial points visited in SEARCH and POLL step. If an unfiltered mesh point was found, decrease the mesh size. Otherwise increase the mesh size and update the filter.

- FILTER UPDATE Let $\mathcal{F}_{k+1}$ be the union of $\mathcal{F}_k$ with all infeasible unfiltered points found during the SEARCH and POLL step. Return to the SEARCH step.

Above algorithm iterates until satisfactory reduction in the objective function is detected and constraint violation is zero.

B. Mesh Adaptive Direct Search Method

One of the drawbacks in the GPS is the restriction to a finite number of poll direction. To introduce more dense set of polling direction in $\mathbf{R^n}$, we adopt the idea of Charles'[38] and construct the variant LTMADS(Lower Triangular Mesh Adaptive Direct Search) to get a set of poll direction. Unlike the GPS, it introduce a poll size parameter Δ_k^p in addition to the mesh size parameter Δ_k^m, and it represents the distance between the incumbent mesh point and the poll points. In GPS, $\Delta_k^m = \Delta_k^p$, but in the MADS, Δ_k^p is set to be larger than Δ_k^m allowing more direction in the frame around the incumbent point.

The graphical explanation about update of mesh size and poll size is shown in Figure 2

Let Δ_k^m and Δ_k^p be the mesh and poll size parameter at the kth iteration respectively. Choose $\Delta_0^m = 1$, $\Delta_0^p = 1$ to be the initial mesh and poll size parameters, and updating rules for these parameters are following.

$$\Delta_{k+1}^m = \begin{cases} \frac{\Delta_k^m}{4} & \text{if } x_k \text{ is a minimal frame center} \\ 4\Delta_k^m & \text{if an improved mesh point is found, and if } \Delta_k^m \leq \frac{1}{4} \\ \Delta_k^m & \text{otherwise} \end{cases}$$

The poll size parameter corresponding to the mesh size can be determined by the positive basis matrix D_k. If the minimal positive basis construction is used, then $n\sqrt{\Delta_k^m}$.[39] If the maximal positive basis constructio is used, then $\sqrt{\Delta_k^m}$.

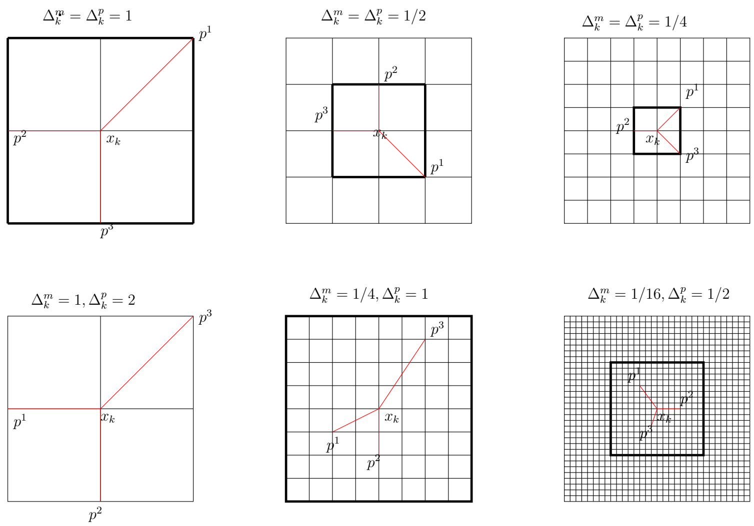


Figure 2. Illustration of the mesh size and the poll size, taken from the paper [39]

The positive spanning set D_k contains a positive basis of $n+1$ or $2n$ as follows.

GENERATION OF THE POSITIVE BASIS D_k

- BASIS CONSTRUCTION: Let B be a lower triangular matrix where each term on the diagonal is either $\pm\frac{1}{\sqrt{\Delta_k^m}}$, and the lower components are integers in the open interval $\left(-\frac{1}{\sqrt{\Delta_k^m}}, +\frac{1}{\sqrt{\Delta_k^m}}\right)$ randomly chosen with equal probability.
- PERMUTATION OF LINES AND COLUMNS OF B: Let $\{i_1, i_2, ..., i_n\}$ and $\{j_1, j_2, ..., j_n\}$ be random permutations of the set $\{1, 2, ..., n\}$. Set $d_p^q = B_{i_p, j_q}$ for each p and q in $\{1, 2, .., n\}$.
- COMPLETION TO A POSITIVE BASIS:
  - A minimal positive basis: $n+1$ directions.
    Set $d^n + 1 = -\sum_{i=1}^{n} d^i$ and let $D_k = \{d^1, d^2, ..., d^{n+1}\}$.
    Set the poll size parameter to $\Delta_k^p = n\sqrt{\Delta_k^m} \geq \Delta_k^m$.
  - A maximal positive basis: $2n$ directions.
    Set $d^n + 1 = -d^i$ for $i = 1, 2, ..., n$ and let $D_k = \{d^1, d^2, ..., d^{n+1}\}$
    Set the poll size parameter to $\Delta_k^p = \sqrt{\Delta_k^m} \geq \Delta_k^m$

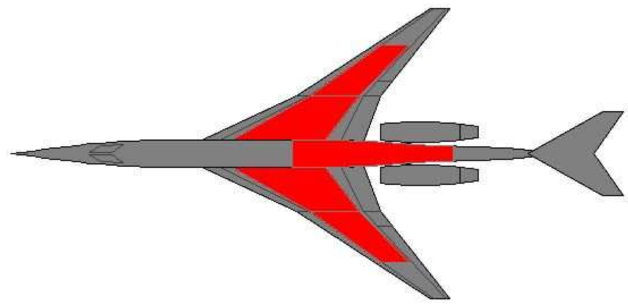
With the more flexibility in the refining direction as explained previously, MADS can provide faster convergence than the GPS.

III. PASS CONCEPTUAL DESIGN TOOL

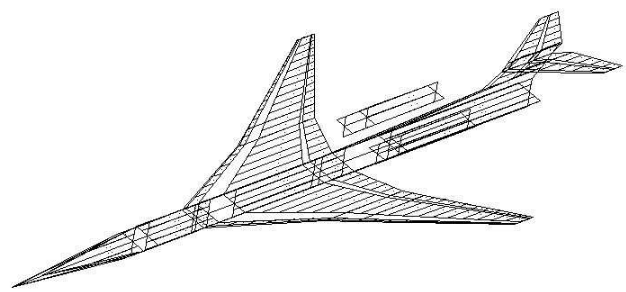
PASS (Program for Aircraft Synthesis Studies), an aircraft preliminary design tool created by Desktop Aeronautics, Inc., was used to generate all of the designs presented in this work. PASS uses a simplex method and an integrated set of predictive modules for all of the relevant disciplines in the design (including mission performance) to generate optimized designs that satisfy a number of imposed constraints.

PASS uses a graphical user interface to explore the results of each of the participating disciplines in a design. In addition, the same interface can be used to define the design optimization problem. Design variables, objective functions and constraints can be setup using any of the relevant parameters and functions that are used in each of the disciplinary modules. A view of two of the various aircraft models (fuel tank arrangement and vortex lattice mesh) that are used by PASS can be seen in Figure 3 below.

Incorporating PASS into the analysis allowed for the evaluation of all aspects of mission performance, thus providing a balanced configuration not just limited to meeting some singular performance goal, but also capable of achieving field length, climb gradient, and cabin constraints (for example) required for a realistic aircraft design. Some of the most relevant capabilities of PASS for this work are briefly summarized below.



(a) Fuel tank layout.



(b) Vortex-lattice model.

Figure 3. Two different views of the configuration within PASS.

A. Drag Estimation

Lift- and volume-dependent wave drag, induced drag and viscous drag are evaluated at key mission points. Inviscid drag is estimated using linearized methods. The viscous drag computation is sensitive to Reynolds number and Mach number, and is based on an experimentally derived fit. Special attention is paid to transonic drag rise, with numerous points being sampled up to and through Mach 1. The analysis detail is of a level that allows configuration tailoring to minimize drag during supersonic cruise (i.e. the use of the area rule is contemplated.) It must be noted that the drag predictions in this module have been constructed so that they can be expected to be a reasonable upper bound in performance if sufficient detailed design work is carried out to properly re-twist and re-camber the wing at the conclusion of the design.

B. Weights and CG

Component weights are based on available data for modern business-jet class aircraft. Wing weight is estimated based on a bending index that is related to the fully stressed bending weight of the wing box coupled with a statistical correlation. The weights of tail surfaces are similarly determined. Fuselage weight is based on both the gross fuselage wetted area and the use of a pressure-bending load parameter.

The CG location is computed based on typical placements and weights of the various aircraft components; CG movement during the mission due to fuel burn is also computed based on the fuel tank layout and the ability to transfer fuel between tanks is also used to aid in the trimming of the configuration throughout its mission.

C. Propulsion

Engines are typically modeled by sampling a manufacturer's deck at numerous Mach numbers and altitudes and constructing a fit. For this study, a generic deck was created and hand-tuned to give performance of a level achievable by available, mature technology, low-bypass turbofan engines.

D. Low-Speed Analysis

Low-speed stability and trim are computed using a discrete-vortex-lattice method. This data is then used to predict such things as the balanced field length (BFL) for the aircraft, stability derivatives and estimates for tail incidences at critical low-speed points (take-off rotation, for example.)

E. Mission Analysis

The mission analysis routine ties together all the various tools in PASS to run an aircraft through a typical flight and evaluate its overall performance. The key points analyzed are the takeoff run, takeoff rotation, 2nd segment climb, subsonic climb to acceleration altitude, subsonic-to-supersonic acceleration, supersonic climb to initial cruise altitude, cruise and landing. In our work in this paper, only the cruise condition benefits from enhanced computations for the aircraft performance in the form of multi-fidelity response surface approximations for the C_D of the complete aircraft.

F. Optimization

PASS combines with a non-gradient based optimizer for configuration studies using genetic algorithm Given some variables, the optimizer will minimize an objective function subject to constraints. The variables, constraints and objective are all user-defined. Typically, the optimizer will be tied to the mission analysis computation. Constraints usually consist of performance goals such as range and balanced field length. Additional constraints to ensure a viable aircraft in the eyes of the FAA may also be imposed, to ensure, for instance, that the aircraft will climb out at the minimum 2.4% gradient stipulated by FAR regulations. Details of the optimization problem formulation are discussed in following sections.

G. Design Configurations

The baseline design, from which all work started, was created using the default PASS analysis modules which are based on classical supersonic aerodynamics and vortex-lattice methods. Subsequent designs are also created with two significant differences: 1) the inviscid aerodynamic drag prediction module in PASS was replaced by the response surface fits created using our multi-fidelity approach, and 2) once a candidate configuration was generated, an adjoint-based wing twist and camber optimization was run to further improve the performance at the cruise condition.

Unlike in our previous work[20] the ground boom calculation module was not used to guide the designs in any way. We will attempt to include sonic boom considerations into the design procedure at a later time.

It is worth noting that in the designs presented in this paper, no assumptions of future technology have been made. All designs use models of existing propulsion plants, materials, and systems that can be incorporated into an actual design today.

IV. AERODYNAMIC ANALYSIS : PASS and A502

the linearized panel code A502/Panair

The complete procedure is as follows. Firstly, a parameterized geometry is represented using a collection of surface patches. These surface patches can be used directly with A502/Panair or can serve as the geometric description for an unstructured tetrahedral mesh generated automatically by the Centaur software. Our geometry kernel, AEROSURF, generates multiple variations of this baseline configuration as required by the response surface construction tool.

linearized panel code A502/Panair calculates the surface pressure distributions and predicts both the C_L and C_D and the near-field pressures which can then be propagated to obtain ground boom signatures in the case of ground boom minimization problem.

A. Geometry Representation

High-fidelity MDO requires a consistent high-fidelity geometry representation. In general, the geometric shape of an aircraft can be defined by an appropriate parameterization of the geometry. This parametric geometry kernel is available to all of the participating disciplines in the design so that both cost functions and constraints can be computed using the same geometry representation. Details of our CAD-based geometry engine (AEROSURF) have been presented earlier[20] and will not be discussed further here.

B. Three Dimensional Linearized Panel Method, A502/Panair

The A502 solver, also known as Panair,[31,6] is a flow solver developed at Boeing to compute the aerodynamic properties of arbitrary aircraft configurations flying at either subsonic or supersonic speeds. This code uses a higher-order (quadratic doublet, linear source) panel method, based on the solution of the linearized potential flow boundary-value problem. Results are generally valid for cases that satisfy the assumptions of linearized potential flow theory - small disturbance, not transonic, irrotational flow and negligible viscous effects. Once the solution is found for the aerodynamic properties on the surface of the aircraft, A502 can then easily calculate the flow properties at any location in the flow field, hence obtaining the near-field pressure signature needed for sonic boom prediction is required by the design. In keeping with the axisymmetric assumption of sonic boom theory, the near-field pressure can be obtained at arbitrary distances below the aircraft.[34]

Although two different flow solver modlues are available for flow calculation, we did not include Euler analysis tool in this study. However it is important to verify the similarities in the solutions provided by each of the flow solvers. As mentioned in the previous section, the drag polars for the baseline configuration using the available flow solvers will be compared in a later section. The panel topologies for the same baseline configurations presented earlier are shown in Figures 4 below.

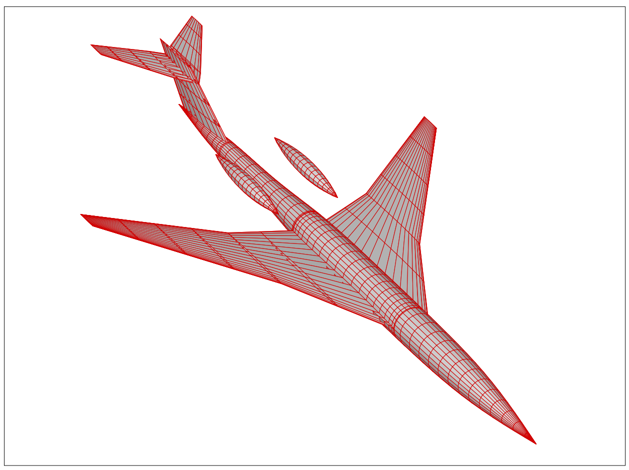


Figure 4. Surface panel representation of complete supersonic jet

Figures 5 shows the the pressure distribution on the surface. For this particular case (Mach number and C_L) the validity of the panel code solutions is comparable to the Euler calculation and the flow patterns that can be observed are similar to those from the Euler results.

The nacelles are represented in A502 not by their true geometry, but instead, by an equivalent area representation that leads to differences in the predicted flow patterns. This is one of the shortcomings of the linearized panel code that can be overcome, at significant computational cost, by using the Euler solver.

V. MULTI-FIDELITY RESPONSE SURFACE GENERATION

As briefly addressed in the introduction, the design space related to the ground boom is known to be noisy and discontinuous, and to have multiple local minima. To demonstrate those characteristics of the design space, Figure 8and 9 shows the two dimensional design space considering two design variables: fuselage radius at 7.5 % and the radius at 11.25 % of the body length are the design variables. Multi-block structured inviscid Euler solver was used to generate these design surface. Design surface for the drag coefficient is rather smooth over the entire domain and easy to approximate. However the design surface of the for the ground boom is not smooth and shows discontinuity. Noise level of the ground boom signature without rise time modification is the objective function in Figure 9

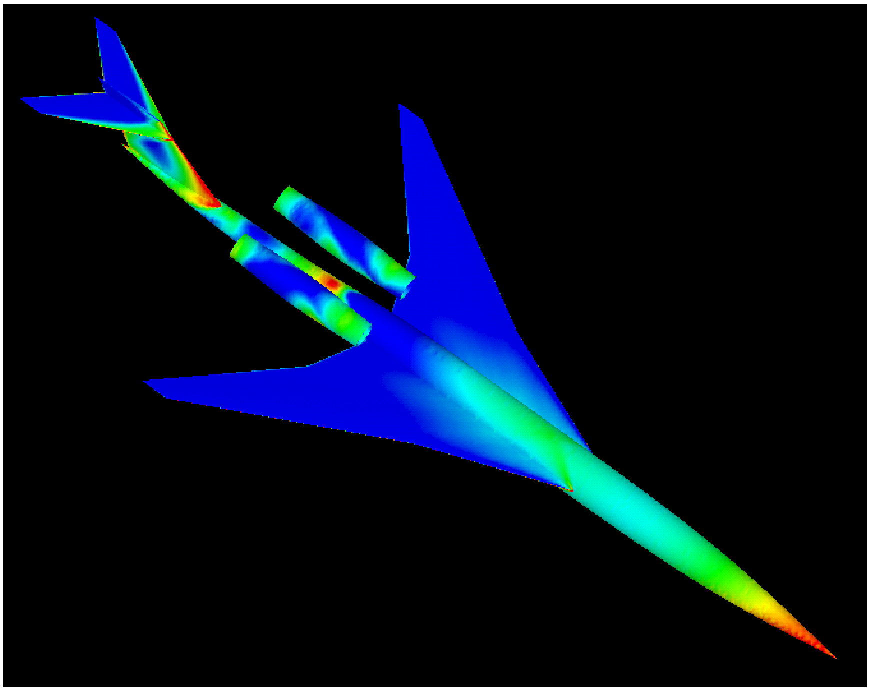
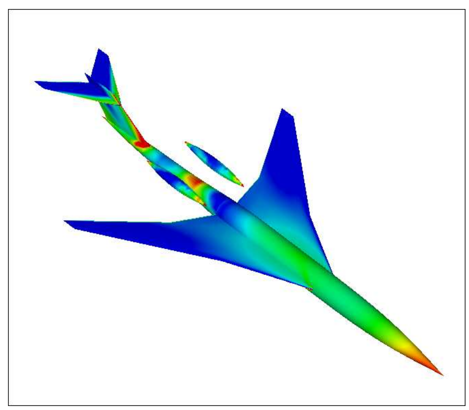
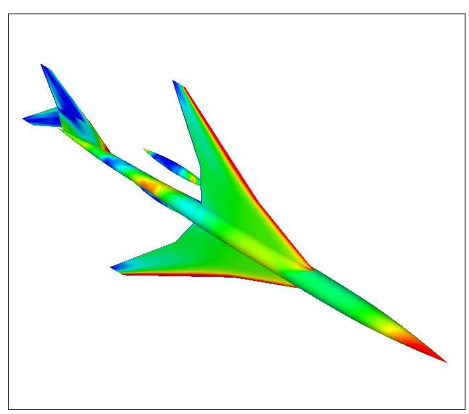


Figure 6. Upper surface pressure distribution for full baseline configuration using AirplanePlus Euler calculation.

Considering the design surface shown above is two dimensional and our study in this paper involves 16 design variables, the complexity of the boom space is becoming higher. Therefore gradient-based optimizer can not be applied to the discontinuous regions and the gradient-free optimizer is required. A large number of function evaluation required for a global optimizer makes the use of response surface critical. The accuracy of the response surface is the key issue in predicting the correct objective functions and the constraints. Now that we have described the two basic tools used to create supersonic designs, PASS and A502, it is necessary to explain the procedure we have used to integrate them into a single analysis and optimization capability. The concept is straightforward: if the multi-fidelity analysis capability can be used to create response surfaces for the drag coefficient, C_D, and ground boom (perceived noise level at the ground without considering rise time modification of the signature), the corresponding low-fidelity modules in PASS can be replaced by these response surface fits. This makes for a remarkably simple integration problem and also provides us with the ability to predict the ground boom signature. The baseline version of PASS is unaware of the actual wing sections used and assumes that, whatever the sections are, they have been adjusted in such a way that the camber and twist distributions are optimal (in the sense that they get close to elliptic load distributions in both



(a) On upper surface



(b) On lower surface

Figure 5. Pressure distribution on the surface

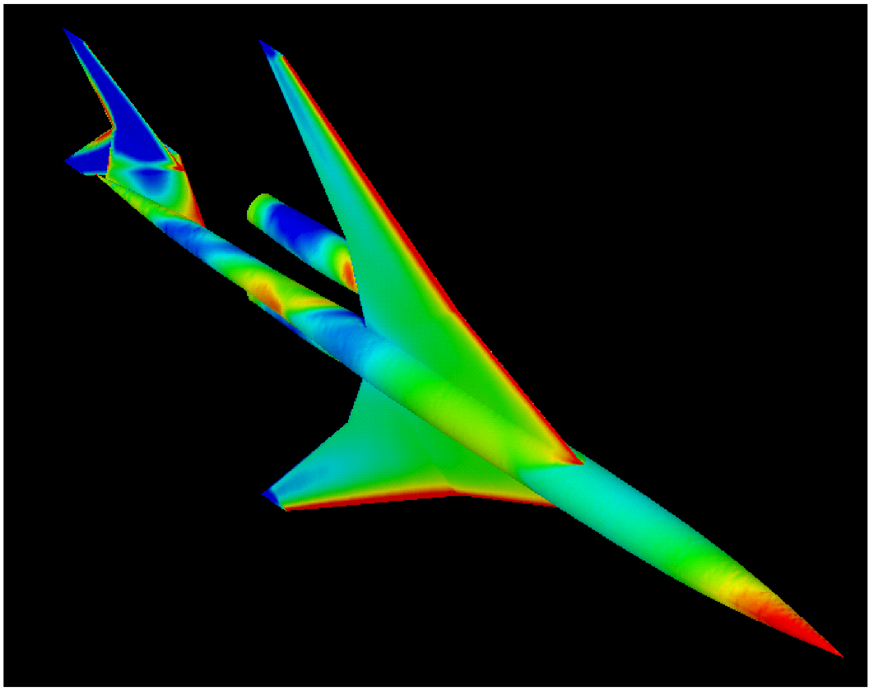


Figure 7. Lower surface pressure distribution for full baseline configuration using AirplanePlus Euler calculation.

the spanwise and streamwise directions). PASS can then be used to generate optimized results and the outcome of the optimization can be analyzed using the high-fidelity tools to ensure that the response surface fits provide accurate representations of the true high-fidelity responses. The level of accuracy in the response surface representation depends greatly on the number of high-fidelity calculations that are used. Since we are trying to minimize this number, we will undoubtedly incur some errors in the fits. The validity of these fits will be assessed by direct analysis of the resulting optimized designs using A502 analysis.

Our multi-fidelity approach to the construction of the response surface fits relies on a hierarchy of two different aerodynamic analysis modules. However the PASS does not have a module to predict the ground boom signature, the fit for the ground boom is constructed mainly by the A502 analysis.

1. PASS internal analysis based on classical aerodynamics.
2. A502/Panair supersonic linearized panel code.

In order to obtain response surface fits of the highest fidelity one could carry out a large number of A502 solutions and fit the resulting data. Unfortunately, for large dimensional design spaces (we will be using a total of 16 design variables and more at other works), accurate fits require a large number of function evaluations. This is particularly true in our case since the ranges of variation of each of the design variables will be rather large.

The main objective in this section is to generate response surface fits of the same quality/accuracy that would be obtained by evaluating the A502 solutions only or Euler, but at a much reduced cost. We accomplish this by relying on a fundamental hypothesis that will be tested later on: *the higher fidelity tools are only needed in small regions of the design space where the lower fidelity models have exhausted their range of applicability.* This is bound to be true as it is the premise upon which aerodynamic design has been predicated for the last 50 years: aerodynamicists and engineers use the fastest tools for a specific purpose (when they are known to work well) and switch to more time-consuming, expensive tools only when they are needed. For example, in supersonic design, classical equivalent area concepts and linearized panel codes can provide very accurate results as long as non-linear effects (such as transonic flows in the direction normal to the leading edge of the wing) are not present and viscosity does not play a dominant role in the solution of the flow.

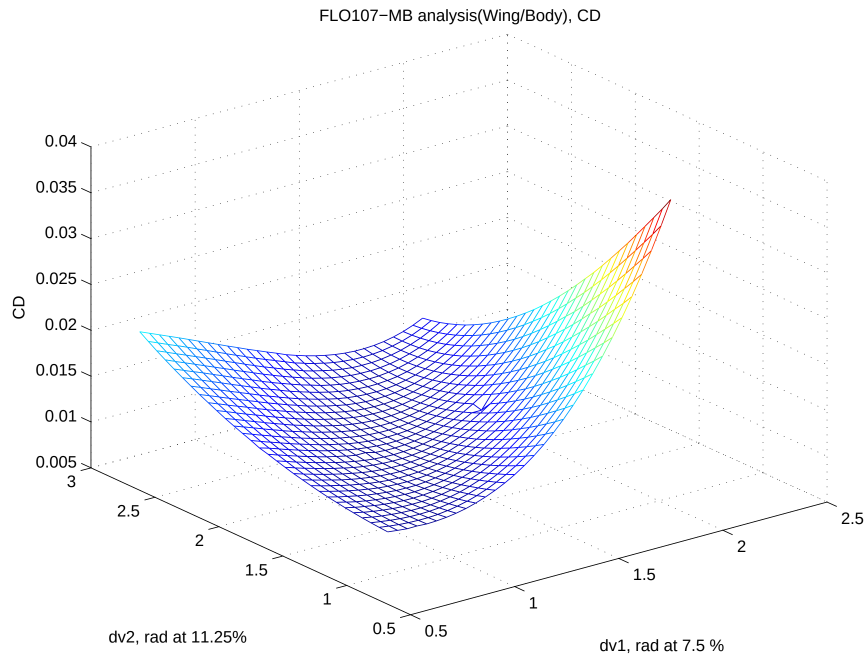


Figure 8. Design surface of drag coefficient with two design variables

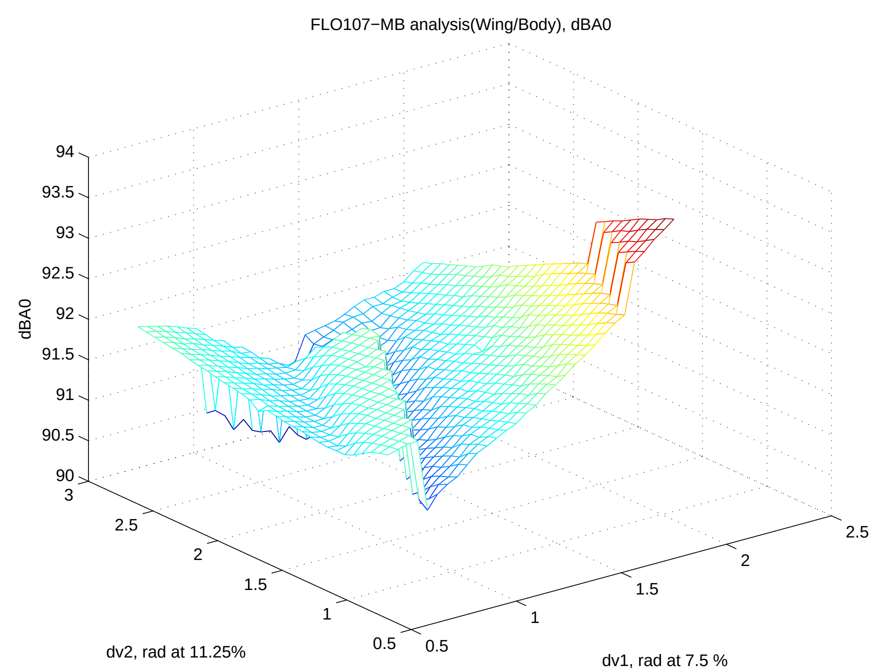


Figure 9. Design surface of noise level with two design variables

With this in mind, we have used the following three-step procedure to create the response surfaces used in this work. All databases of candidate designs are obtained by populating the design space using a Latin Hypercube Sampling (LHS) technique.

1. Run a large database of candidate designs ($> 1,000$) using the aerodynamics module in PASS. Each evaluation takes less than a second to compute on a modern workstation (Pentium 4, 3.2 GHz). This evaluation also flies each aircraft through the mission and returns a measure of the infeasibility of the design (an L-2 norm of the constraint violations.) Those designs that are found to significantly violate the requirements/constraints of the mission are removed from the database and are no longer considered in the response surface creation.

2. Run the remaining database of candidate designs (≈ 300) using the A502/Panair solver. Each evaluation requires about 10 seconds of CPU time on the same modern workstation.

3. For the C_D, a baseline quadratic response surface fit (using least squares regression) is created obtained with PASS. The error between the values of the A502 evaluations and the predictions of these quadratic fits is approximated with a Kriging method, and the resulting approximation is added to the baseline quadratic fits. For the perceive noise level for the ground boom, PASS does not have the capability to predict the ground boom and the Kriging fit for the ground boom signature obtained by A502 analysis will be used.

In sum, the response surfaces provided to PASS are the addition of the quadratic fits based on the A502/Panair results and the Kriging fits of the error between the A502 solutions and those quadratic fits.

This multi-fidelity procedure has, to some extent, the flavor of Richardson's extrapolation in that it recursively uses results from different fidelities to arrive at a final answer/fit. It also has an adaptive nature to it, as results from the higher fidelity models are only evaluated in areas of the design space where the lower fidelity models are found to be insufficiently accurate. If the hierarchy of models is chosen in such a way that the areas where the lower fidelity models fail are small compared with the size of the design space, then the procedure described above should be quite effective in producing results that are of nearly high-fidelity over the *entire* design space. Our experience shows that this is the case for aerodynamic performance: the PASS aerodynamic module is quite good at predicting the absolutely best wing (lower bound estimate on the C_D) that could be produced if considerable design work were done on the configuration (potentially using adjoint methods and a high-dimensional shape parameterization). However, it is unable to predict some of the finer details of aerodynamic performance and certainly fails when transonic effects are present. A502/Panair is also unable to deal with transonic flow effects but produces more realistic results than the PASS analysis as the actual geometry of the configuration is truly accounted for.

VI. OPTIMIZATION TOOL: Genetic Algorithm

Genetic Algorithms (GAs) are one of the popular global search algorithms based on the evolutionary ideas of natural selection and genetic operations. They have been successfully used for a shape optimization of supersonic jets in the previous work.[17] One of the major drawbacks is the expensive computational cost related to a large number of funciton evaluation, but the use of accurate surrogate model enhances the convergence. In our work, the algorithm of Srinivas and Deb,[43] Non-dominated Sorting Genetic Algorithm II (NSGA II) is used. This algorithm tackles the multi-objective problems with the improvement in computational complexity by using non-dominated-sorting and sharing. Even though it can handle multi-objective optmization, we treat our problem as a single objective problem for a minimum MTOW and ground boom noise level is considered as one of the constraints in addition to the mission requirements.

VII. RESULTS

A. Baseline Configuration: Standard PASS Optimization

For subsequent design work, an optimized baseline geometry was generated by running the standard version of PASS for a mission with the performance objectives summarized in Table 1. Mission requirements and geometric constraints for the baseline configuration were based on numbers that were felt to be representative of current industry interest. The value of the MTOW is the result of the optimization as this was the objective function of the design. As mentioned before, in an effort to generate an aircraft achievable using current levels of technology, advanced technology assumptions were kept to a minimum.

Cruise Mach	1.6
Range	4,000 nmi
BFL	6,500 ft
Minimum static margin	0.0
Alpha limit	15°
MTOW	96,876 lbs

Table 1. Performance requirements for optimized baseline configuration.

The values of the design variables for the resulting baseline configuration (which are also used as starting points for subsequent designs) are provided in Table 2. Note that the values highlighted in red were not allowed to vary during this initial optimization. In addition to these variables, 6 variables representing the radii of fuselage stations located at 3%, 7.5%, 11.25%, 62.5%, 75%, and 87.5% of the fuselage length were added to allow for performance improvements and to maintain cabin and cockpit compartment constraints. Finally wing section changes were allowed at three defining stations. The twist at the root/symmetry plane section, the leading edge crank section and the tip section were allowed to vary.

Note that the allowable ranges for all of the design variables (for this baseline configuration and all subsequent designs) were rather large, being at least $\pm 30 - 40\%$ of their baseline values. This large range of variations allows for a more complete design space to be searched but also makes the job of both the optimization algorithm and the response surface fitting techniques more complicated. The values of the leading and trailing edge extensions in the Tables are normalized by the trapezoidal wing root chord. The location of the wing root leading edge is normalized by the fuselage length and is measured from the leading edge of the fuselage. Both the vertical and horizontal tail areas are normalized by S_{ref}.

B. PASS Optimizations Using Response Surface Fits

Changes were made as necessary to the PASS code to incorporate the C_D fits. For more details of how this was accomplished, the reader is referred to.[20]

1. *1st Design iteration*

In this section we present the results of the optimization that used the version of PASS that had been enhanced with the response surface fits created with our multi-fidelity approach. The focus was on minimizing the MTOW of the aircraft while meeting all of the mission requirements.

Figure 15 shows the filter at initial condition. Response surface was constructed by the points randomly located in the design space by LHS, and they are not on the initail mesh. Several points around initial baseline and some random points on the mesh consist of initial points on the mesh. The solid line is the filter at initial condition. The baseline shows the least constraint violatioin.

After the first GA run for an optmization, the least infeasible point is shown in Figure 15. MTOW is reduced by 1553.43 lb and constraint violation is decreased at the same time. Comparison is shown in Table 3

Figure 10 and 11 compares the surface pressure distribution between baseline configuratin and the 1st optmized configuration. Changes in the wing planform and fuselage radius are shown in Figure 12.

2. *2nd Design iteration*

The optimized point (the 1st incumbent point) in the filter shown in Figure 15 dominates all other points in the initial filter. Therefore search step was successful, however the mission constraint violation can not be ignored. In order to update the response surface and the current mesh, the current incumbent point is included in the response surface. However original response surface was constructed with the number of about 300 initial points bye A502, adding only one point does not improve the quality of the response surface. Therefore about 50 more points including the points on the postitive basis directions, are included in updating the response surface.

Wing and Tail Geometry

Wing reference area (S_{ref})	$1,078$ ft^2
Wing aspect ratio (AR)	4.0
Wing quarter-chord sweep (Λ)	53.35°
Wing taper	0.15
Wing dihedral	3°
Leading edge extension	0.278
Trailing edge extension	0.197
Break location	0.4
Location of wing root LE	0.294
Root section t/c	2.5%
Break section t/c	3.0%
Tip section t/c	2.5%
Vertical tail area (% S_{ref})	0.125
Vertical tail AR	0.65
Vertical tail Λ	56°
Vertical tail λ	0.6
Horizontal tail area (% S_{ref})	0.6
Horizontal tail AR	2.0
Horizontal tail Λ	56°
Horizontal tail λ	0.3

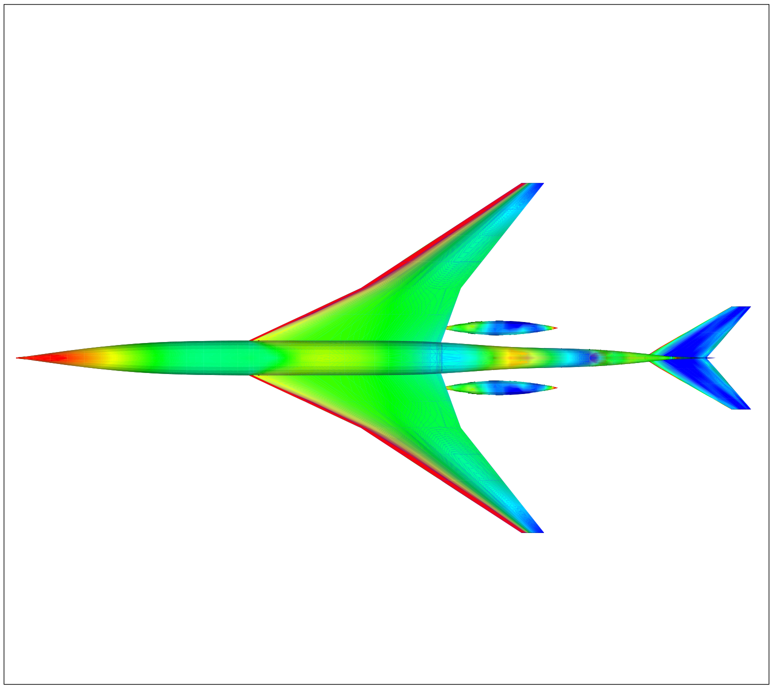
Fuselage Geometry

Maximum fuselage length	125 ft
Minimum cockpit diameter	60 inches
Minimum cabin diameter	78 inches
Cabin length	25 ft

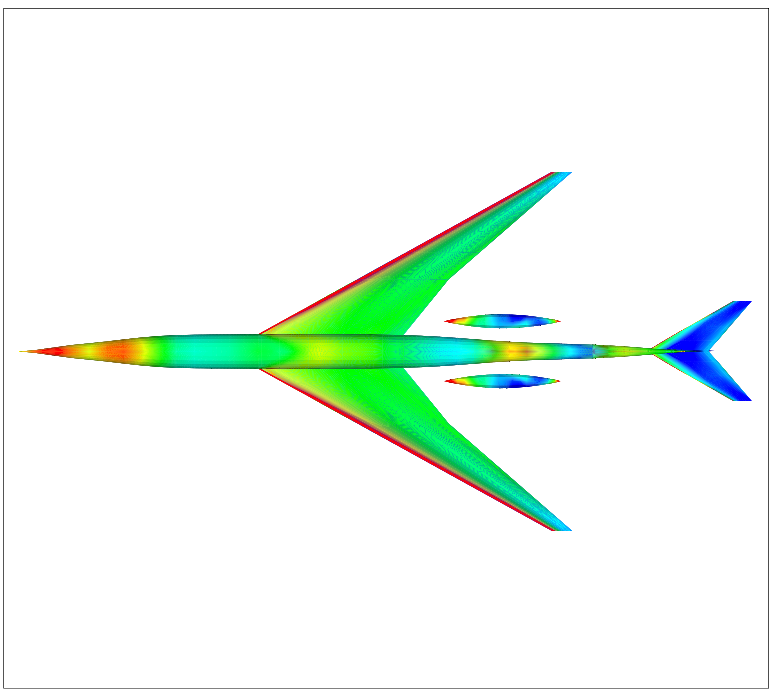
Table 2. Geometric design variables for design optimization and values for baseline design.

	baseline	1st optimized	2nd optimized	improvement (%)
MTOW	96,876 *lb*	95,322	94,714	-2.2317
C_L	0.1323	0.14	0.13963	+5.15
C_D	0.009365	0.00948	0.008503	-9.2
LD	6.62	7.01	16.36	+15
ground boom	92.24	91.3	91.2	-1dB

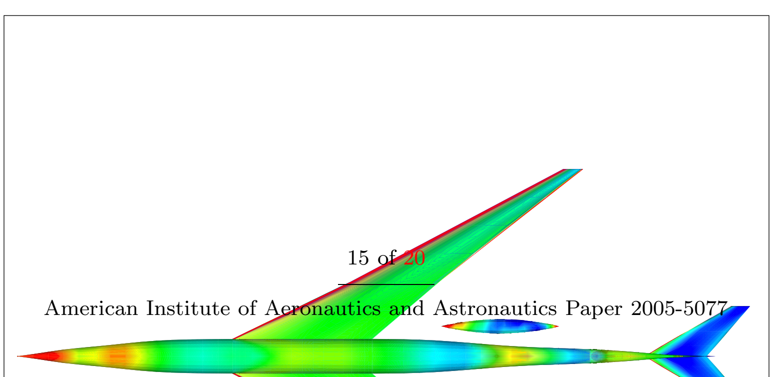
Table 3. Performance requirements for optimized baseline configuration.

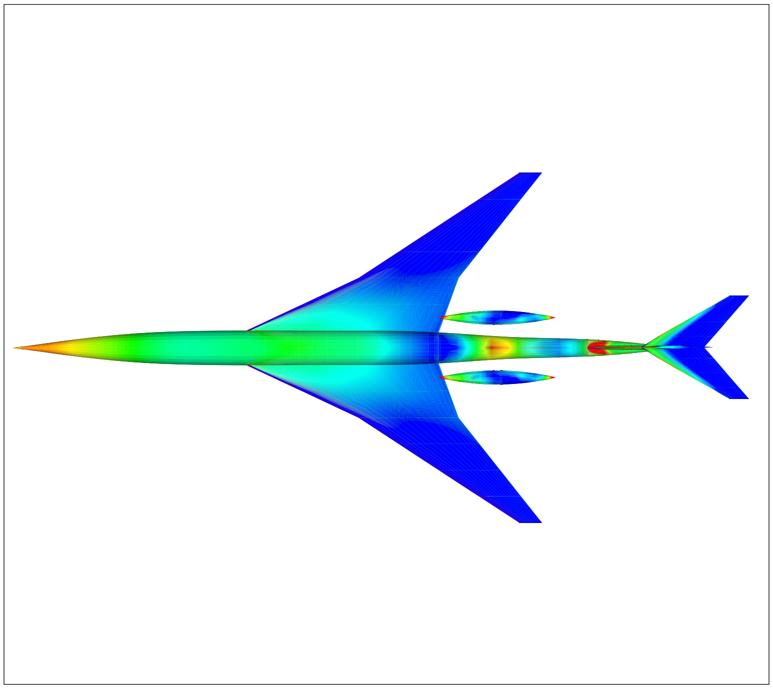


(a) baseline

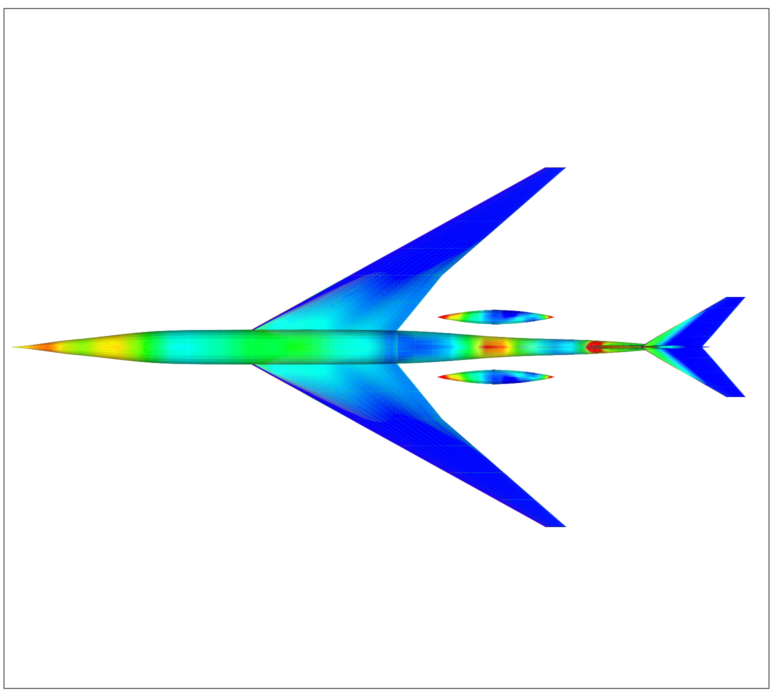


(b) 1st optmized

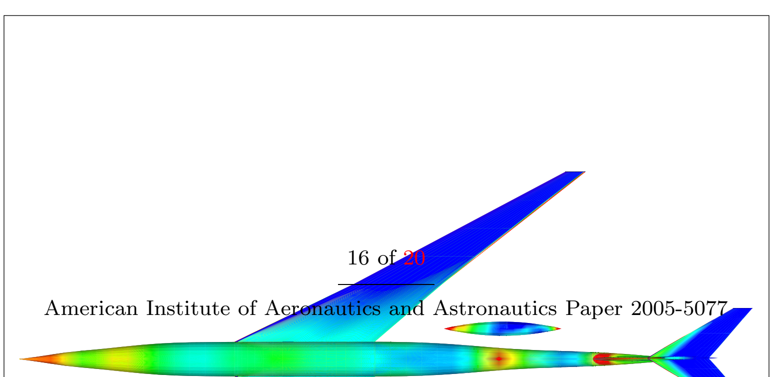


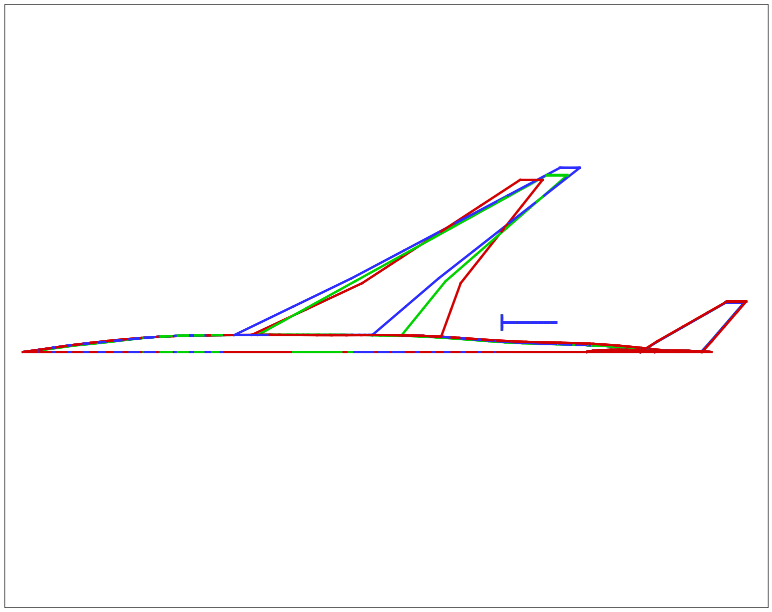


(a) baseline

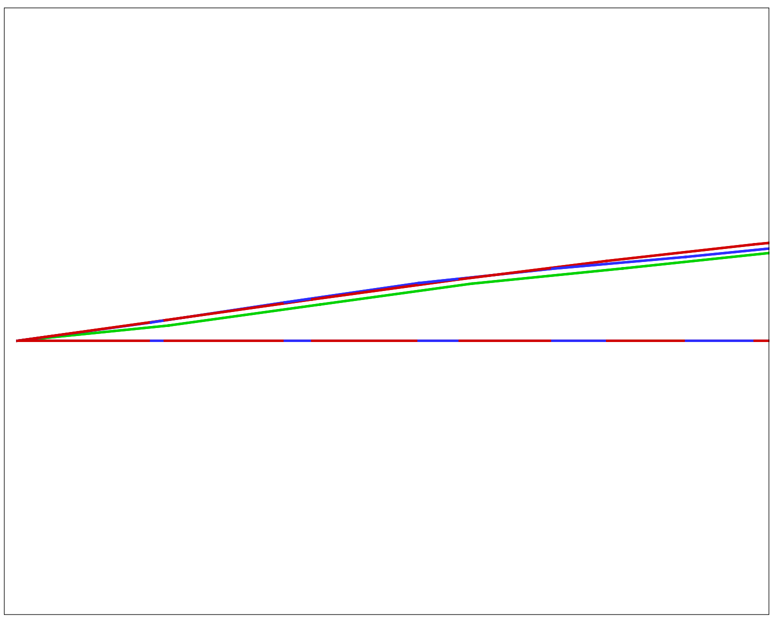


(b) 1st optmized

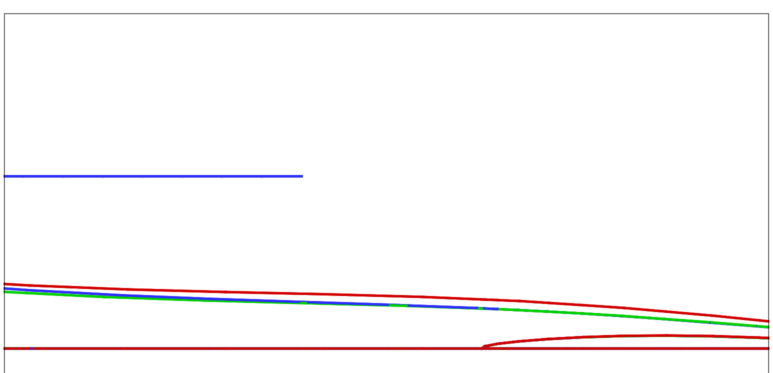




(a) entire view



(b) zoomed in view around nose



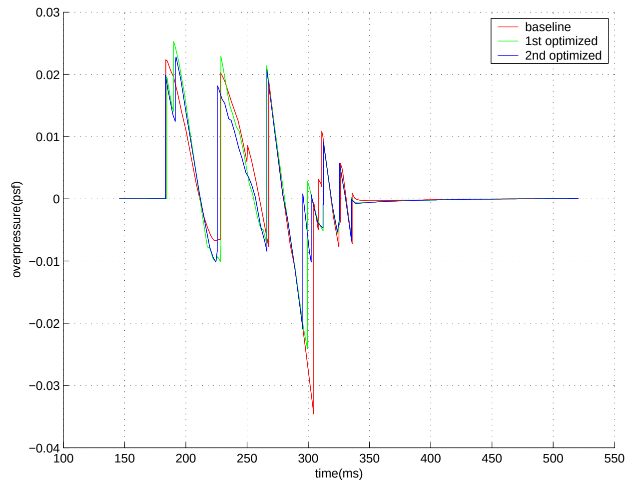


Figure 13. Filters at initial condition and 2 Design iterations

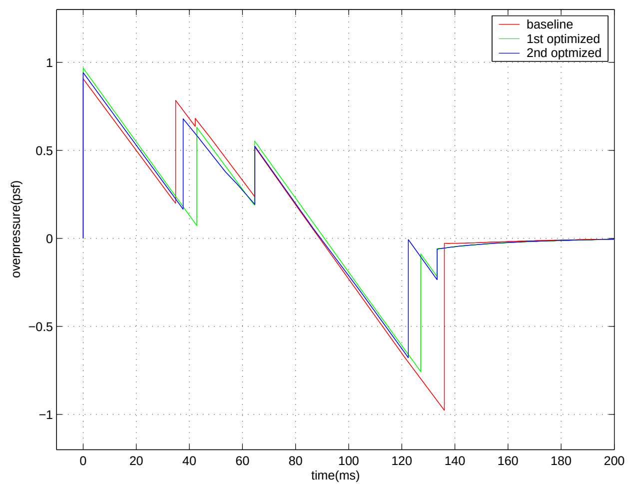


Figure 14. Filters at initial condition and 2 Design iterations

With the updated new response surface, the second GA run is performed. New improved point was found and is ploted in the Figure 15 as the 2nd incumbent point.

VIII. CONCLUSIONS AND FUTURE WORK

Shape optmization of supersonic jets is implemented using the filtered MADS method. After two design iterations, the optmized configuration shows a improved aerodyanmic/boom performance and satisfies most of the constraints. After applying more MADS runs, we can find the better configuration satisfying all the mission requirements.

This work is done with two different fidelity tools, PASS and A502 to show that filtered MADS is applicable in our design problem. Since verification of the methods is shown in this work, higher-fidelity tool, such as inviscid Euler solver, can be employed like in our previous work. This is left for the future work.

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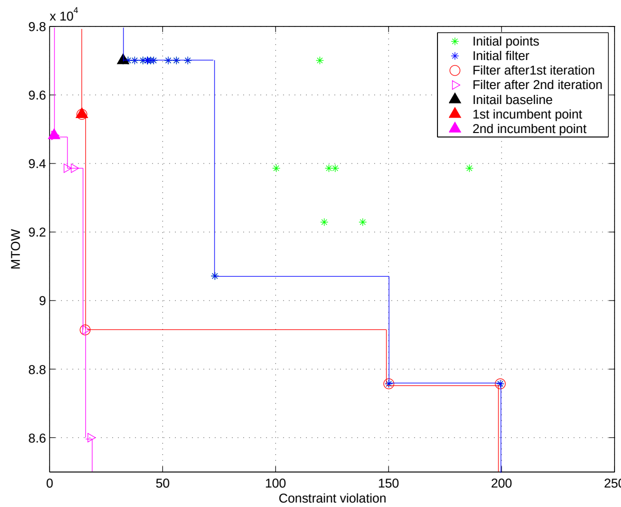

Figure 15. Filters at initial condition and 2 Design iterations

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