AIAA Aviation
22-26 June 2015, Dallas, TX
16th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference

AIAA 2015-3439

A Multi-Fidelity Approach to Quantification of Uncertainty in Stability and Control Databases for Use in Stochastic Aircraft Simulations

Andrew D. Wendorff* and Juan J. Alonso†
Stanford University, Stanford, CA, 94305

Stefan R. Bieniawski‡
Boeing Research & Technology, Seattle, WA, 98124

Stability and control derivatives are of utmost importance in the design of new aircraft. Companies spend millions of dollars trying to certify aircraft that are designed with only basic estimates of these features. This paper outlines a methodology to combine multiple fidelity levels of analysis tools in a stochastic aerodynamic database using surrogate models to represent the aircraft configuration. The vehicle of interest is then simulated through certification maneuvers incorporating a deterministic instance of the database to calculate the probability of meeting the requirements. Deterministic instances of the database are built incorporating a multivariate Gaussian conditioned on previous analysis points necessary for the maneuver. The stochastic database is updated by adding samples found by either minimizing the computational cost for a specified uncertainty or minimizing the uncertainty for a fixed computational cost. This analysis capability is applied to the NASA CRM using a fixed number of trim points to simulate an emergency descent maneuver with a Vortex Lattice Method as the high fidelity information source and conceptual level handbook methods as the low fidelity information source. Output distributions characterized by the mean and variance are generated to estimate the certification probability. This analysis tool provides designers with information on likelihood of certification and the potential to identify areas of the design dominating uncertainty in meeting requirements.

I. Introduction

During the conceptual and preliminary aircraft design phases, there is often a need to estimate the static and dynamic responses of the vehicle. These characteristics are used as a way to estimate the necessary Stability & Control (S&C) requirements that govern control surface location, sizing, and range of motion to generate desired forces and moments. While aerodynamics, propulsion, and weights receive most of the attention in the design of new vehicles, S&C requirements and control surface sizing are relegated to simplistic constraints or sizing based on previous aircraft. It is not until later in the design process where an in-depth understanding of S&C characteristics and their effect on vehicle performance is obtained. To estimate certification requirements, aerodynamic databases (containing force and moment coefficients with respect to flight condition, angular rate, control surface deflection, etc.) of the design need to be constructed and simulations run to determine if the configuration meets the certification requirements. These deterministic databases are typically estimated with aerodynamic information coming from a number of different information sources (including wind-tunnel and flight testing) with varying associated costs, fidelity levels, lead times, uncertainties, and vehicle mass and inertia properties. At present, these different analysis techniques are only applied to a vehicle when the configuration is primarily set and the opportunity to make slight changes in the design that significantly improve the probability of meeting certification

*Graduate Student, Department of Aeronautics and Astronautics, AIAA Student Member.
†Professor, Department of Aeronautics and Astronautics, AIAA Associate Fellow.
‡Technical Fellow, Senior Member AIAA.

requirements has been lost. When readying the 787-9 for testing and certification, Boeing used the 787-8 as a surrogate in order to improve performance predictions over a year before the first 787-9 took flight.[1] In addition to potentially modifying the aircraft design, pushing S&C analysis further forward in the design phase allows control systems to be better integrated into the system enabling additional capabilities and improving performance instead of fixing problems.

Moving S&C analysis techniques further forward in the design process is not as easy as using the same methodologies the same way, just earlier. Despite the fact that there are a multitude of methods to compute data contained in aerodynamic databases, there is no formal procedure to determine, for a desired level of accuracy/uncertainty, which tool to use when, and in what region of the flight envelope. Boeing is currently using the 787-9, where an interaction between the aircraft's maneuver load alleviation system, through the deployment of spoilers, and the first body-longitudinal-bending mode is found to cause additional pinging of the structure, to help in the control design of the 777X.[1] While this case solved a problem before it became an issue for the next configuration, the methodology does not take into account all the different sources of information available about the aircraft and instead relies on using similar aircraft to identify issues. As such, our intent is to construct a mathematical framework capable of determining where a vehicle should be analyzed and with what information source to reduce the uncertainty in meeting the certification requirements for minimum cost.

To create configurations that will robustly meet requirements over vehicle changes in the design process, uncertainty quantification (UQ) techniques are necessary. Implementing certification maneuvers on aircraft models with integrated stochastic databases, which are not currently generated in the design process, necessitates a new analysis approach. We have proposed a potential solution to the construction and sampling procedure of stochastic databases that will work in connection with legacy deterministic analysis tools. We can then identify areas in the aerodynamic database where uncertainty in coefficients results in the greatest likelihood of not meeting certification requirements. Choosing to sample one of the information sources based on cost, outputs desired, and necessary accuracy can then reduce this uncertainty efficiently. This procedure provides an engineer the methodology to estimate the probability the aircraft, as designed, will meet design requirements and the potential to modify the configuration to rectify potential issues before an aircraft is built. In this paper, we cover our nomenclature and general optimization problem architectures in Section II. Section III sets up the problem formulation, connecting the vehicle inputs, information sources, maneuvers of interest, and statistical outputs. The stochastic aerodynamic database construction mechanism and sampling methodology are outlined in Section IV. Our application, including the configuration to analyze along with information sources and the maneuver of interest, is discussed in Section V. The results of our work and comparison to existing analysis methods are in Section VI. Finally, Section VII contains our conclusions and future work.

II. Mathematical Setup

To understand how our methodology fits into the design and analysis of new configurations, we must explain the mathematical notation used in defining our concepts. After describing our nomenclature, we can then formalize the optimization problem into two general setups, minimize uncertainty for a fixed cost and minimize cost for a desired uncertainty level.

A. Nomenclature

When building a mathematical framework, it is important to have a common nomenclature with which to discuss ideas. For this paper, we use f_i to denote an information source with f_0 being the highest fidelity level and f_i where i = 1, 2, ..., m are sorted by decreasing fidelity levels. m is the total number of information sources available. $\mathbf{x}$ will be used for the design space with $\mathbf{x_i}$ denoting locations in the design space sampled at fidelity level i and $\mathbf{x^k}$ marks the sample number k. $\mathbb{R}^r$ is the span of the aerodynamic database with r denoting the number of inputs of interest. For each analysis, it will be assumed, $c_0 > c_1 > c_2 ... > c_m$ where c_i is the cost of generating one data sample at fidelity level i and $\mathbb{C}$ is the total computational budget available. n_i is the number of samples at fidelity level i. As we are interested in potentially obtaining multiple outputs from a single function evaluation $f_i(x_i)$, we will specify deterministic outputs as $f_i^j(x_i)$ where j = 1,...,p and p is the number of outputs of interest. Each information source f_i is not constrained to provide all the desired outputs j, but all outputs can be calculated with f_0. In order to remove the ambiguity between deterministic outputs that might come from computational codes and stochastic results, we will use $y_j = f_j(\mathbf{x})$ as the deterministic results and define $F_j(\mathbf{x}) = P(y_j, \mathbf{x})$ as a function that takes the deterministic information source results and creates a stochastic output at that level of fidelity. To focus on multi-fidelity approaches and combining different fidelity analyses, $F_0(\mathbf{x_0}) = f_0(\mathbf{x_0})$ meaning the highest fidelity level has no uncertainty when sampled. We denote our stochastic databases by $D(\mathbf{x})$ and the deterministic database $d(\mathbf{x})$ is instanced using a sampling procedure $a(D(\mathbf{x}))$. The certification maneuver functions, denoted $g_j(d)$ for the jth maneuver, take a deterministic aero-database and output a set of conditions about the trajectory $\mathbf{z}_j = g_j(d)$. These trajectory outputs are then combined following a UQ algorithm $Q(\mathbf{z}_j)$ to construct output distribution Z. Deterministic functions q and h denote the Quantity of Interest (QoI) for the optimization and both the equality and inequality constraints, respectively. Throughout this paper, we will use lower-case letters for deterministic quantities/equations and capital letters for stochastic versions of the same.

B. Optimization Objectives

Since our primary goal is to efficiently incorporate multiple information sources to manage uncertainty, we must think about how to incorporate new pieces of data. Depending on the ultimate objective, cost or uncertainty, the QoI and constraints change.

1. *Minimizing the Uncertainty in Aircraft Simulation*

In the first conception, the end user desires to minimize some part q of the distribution Z$(\mathbf{z}_p^1, ..., \mathbf{z}_p^k)$ for maneuver p where each $\mathbf{z}_p^k$ is a function of all the samples $\mathbf{x^l}$. q could be the variance in the distribution, a confidence interval, an acceptable error, or another deterministic quantity of interest. $h(Z(\mathbf{z}_t^1, ..., \mathbf{z}_t^{kk}), F_i^j(\mathbf{x_i}))$ denotes a constraint over maneuver t that must be satisfied. Among the possible constraints placed on this optimization procedure is the total computational budget available. This situation is representative of the requirement that our methodology meets the rapid design environment where designs need to be studied quickly and feedback provided because configurations are always changing. The formal mathematical optimization set up for this problem is

$$\begin{array}{ll} \underset{\mathbf{x}^{k+1}}{\text{minimize}} & q\left(\mathbf{x}_{ii}^1, ..., \mathbf{x}_{jj}^k\right) \\ \text{subject to} & h_{iii}(Z(\mathbf{z}_t^1, ..., \mathbf{z}_t^{kk}), F_i^j(\mathbf{x_i})) \leq 0 \quad iii \in \{1, ..., tt\} \\ & \sum_{i=1}^{m} c_i n_i \leq \mathbb{C} \\ & lb_{jjj} \leq x_{jjj} \leq ub_{jjj}, \; jjj \in \{1, ..., r\} \\ & \mathbf{x}^{k+1} \in \mathbb{R}^r \end{array} \tag{1}$$

where tt is the total number total number of constraints.

2. *Minimizing Computational Cost*

The second way we set up the problem uses the constraints to set the uncertainty necessary from the $S\&C$ characteristic of interest with the minimized computational cost necessary to meet the requirement. This formulation is indicative of a slightly later step in the vehicle design process where, instead of trying to quickly analyze as many configurations as possible, the objective is to satisfy all the desired uncertainty requirements by using a set of maneuvers to estimate a total probability the vehicle will be certified. These conditions manifest themselves as constraints (h's) on the performance metric defined by

$$\begin{array}{ll} \underset{\mathbf{x}^{k+1}}{\text{minimize}} & \sum_{i=1}^{m} c_i n_i \\ \text{subject to} & q\left(\mathbf{x}_{ii}^1, ..., \mathbf{x}_{jj}^k\right) - q_{max} \leq 0 \\ & h_{iii}(Z(\mathbf{z}_t^1, ..., \mathbf{z}_t^{kk}), F_i^j(\mathbf{x_i})) \leq 0 \quad iii \in \{1, ..., tt\} \\ & lb_{jjj} \leq x_{jjj} \leq ub_{jjj}, \; jjj \in \{1, ..., r\} \\ & \mathbf{x}^{k+1} \in \mathbb{R}^r \end{array} \tag{2}$$

These two situations do not have to be mutually exclusive, our methodology can be extended to a situation where there is a set computational limit integrated with a required performance characteristic on the output parameter.

III. Problem Formulation

To analyze this type of problem, there are multiple different pieces of information that must be specified. These include:

- Aircraft Characteristics
- Information Sources
- Database Construction Method
- Certification Maneuvers
- UQ Methodology
- Maneuver Approval Measure
- Quantity of Interest

The integration of these different inputs into a mathematical analysis component is shown in Figure 1.

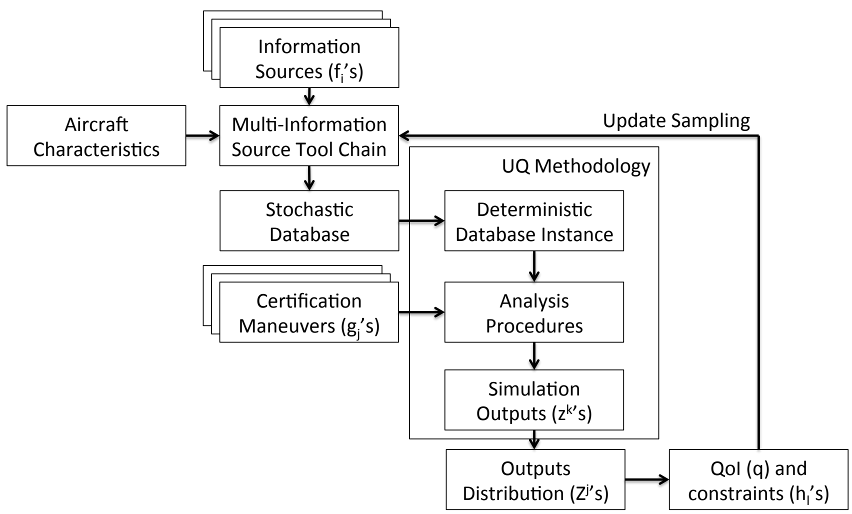


Figure 1: Connection between Components to Analyze Configuration for Certification Maneuvers

A. Aircraft Characteristics

Within our current methodology, we assume that the physical aircraft characteristics are provided. Parameters that would change the aerodynamic characteristics generated from our information sources are considered to be known exactly. The aircraft outer mold line (OML), control surface characteristics, number of engines, and engine maximum thrust are all considered to be fixed in our analysis. We allow other measures of the aircraft such as the total mass, moment of inertia, and idle throttle setting to be specified as distributions. More generally, stochastic variables can be assigned for any attribute where the information source analyses will return the same aerodynamic results. This flexibility is useful in the conceptual design phase, where one aircraft geometry might have different weight estimates based on the set of tools used to analyze the configuration. The designer can thus start to understand how the uncertainty in other analysis components will propagate through to performance estimates and certification probability.

B. Multi-Information Source Tool Chain

Once an aircraft has been created (hypothetically using a conceptual design tool), a number of different information sources can be used to analyze this configuration. We treat the information sources as black-box deterministic codes providing some pieces of aerodynamic data, but not necessarily all required information for simulation of the vehicle. Along with the aerodynamic data, there is uncertainty in each parameter not directly output from the highest fidelity information source and a specific cost to generate the data. For example, the multiple information sources might be Large Eddy Simulations (LES), Reynolds Averaged Navier-Stokes (RANS), and Euler calculations all generating aerodynamic coefficients, where uncertainty in the codes is increasing as the cost of analysis is decreasing. Depending on the assumptions made, we can assume the highest possible fidelity level, in this example LES, will be the "truth" model and provide exact analysis characteristics. This assumption simplifies the problem at present so we can focus on the resource allocation problem while avoiding model-form uncertainty.

C. Aerodynamic Database

After a hierarchy of information sources has been constructed, a mathematical compilation of aerodynamic data for both the best estimate of each parameter and a quantification of the uncertainty needs to be built. At present, we only know of aerodynamic databases that contain an estimate of the coefficients without this measure of uncertainty, refered to as a deterministic database. One of the major contributions in this paper is the mathematical construction of the stochastic database, which adds the uncertainty in the parameters of interest. The methodology we use in the initial construction of the stochastic database by combining multiple information sources along with a way of instancing deterministic samples of the stochastic database is covered in detail in Section IV.

D. Flight Certification Maneuver

Beyond specifying the aircraft attributes and analysis information sources, a number of certification maneuvers ($g_j^{\prime}s$) of interest should be specified. These maneuvers would be considered limiting cases that cover potential problems further in the design process and flight conditions, where control systems could improve vehicle performance or certification probability. As part of this process, the methodology for how to analyze the maneuvers must be specified. Existing ways to simulate the trajectories include trim point analysis, dynamic simulations,[2] six-degree-of-freedom computations,[3] and reachability for linearized systems[4] among others. Specifying tests for aircraft control systems could be incorporated into these maneuvers of interest. Our methodology is flexible enough to incorporate any deterministic database analysis capability considered by an engineer.

E. Uncertainty Quantification Methodology

When both the stochastic aerodynamic database, constructed by combining the multi-information source data for the vehicle, and the flight maneuvers of interest have been specified, a UQ methodology will combine this information. The certification maneuver expects deterministic inputs, in general, as these capabilities calculate how one specific instance of the vehicle behaves. For example, Monte Carlo sampling the stochastic database creates different deterministic instances of the vehicle that simulate the certification maneuver. This analysis requires the smallest number of assumptions about the distribution of the stochastic database, but is also the least efficient. If the stochastic database contains variables characterized by known probability distributions (normal, uniform, exponential, etc.), then polynomial chaos expansion (PCE) can significantly reduce the number of samples necessary to quantify the uncertainty in the function (in our case, maneuver) of interest.[5] Stochastic collocation methods could also be used in this methodology. Determining the appropriate UQ methodology and implementing those techniques into the analysis structure is an ongoing research topic that will be more fully studied in future research.

F. Measure of Certification Approval

Using some UQ scheme, a set of vehicle trajectories are created based on the instances of the deterministic database. This set of trajectories $(z^1, ..., z^k)$ needs to be condensed to determine if the certification requirement is met. For a takeoff requirement, this condition could be a binary result calculating the elevator deflection necessary to pitch the aircraft. If the elevator deflection is too large, then the aircraft fails. Otherwise, it passes. The measure could also be a second segment climb gradient, where the gradient is an output distribution that can be categorized by a mean and variance. Every maneuver needs a measure to determine if the the vehicle satisfies the requirements. This comparison of the trajectories can be combined in the $h_i(Z(\mathbf{z}_j^1, ..., \mathbf{z}_j^k)$ in the optimization formulation to incorporate confidence levels of the control system and other limiting characteristics.

G. Quantity of Interest

At the same time, we must set the quantity of interest (QoI) $q(\mathbf{x})$ as the objective that is to be minimized is our first formulation. Within our methodology, we do not vary the vehicle, but instead try to minimize the uncertainty in certification trajectories. Going back to our example on calculating second segment climb gradients, an inappropriate QoI would be to maximize the mean climb gradient. Instead, we would want to focus on a measure of the uncertainty, possibly the variance, that tells whether the aircraft as presently designed meets the FAR requirement of the second segment climb gradient. This QoI could then be combined with a constraint on other maneuvers, maybe the elevator deflection angle during takeoff, to populate the optimization problem defined in Section II above.

IV. Stochastic Aerodynamic Databases

One of the major emphases of this paper is the construction and use of a stochastic aerodynamic database. We are not currently aware of other literature building or incorporating stochastic databases into aircraft analysis. Our approach for development and sampling of these databases is described below.

A. Construction of Stochastic Database

To integrate different information sources into a stochastic database, we incorporate three steps:

1. Determine locations of information sources $\mathbf{x}_\mathbf{i}^\mathbf{k}$ to sample
2. Combine $f_i^j(\mathbf{x}_\mathbf{i}^\mathbf{k})$ of each information source into $F_i^j(\mathbf{x})$to span $\mathbb{R}^r$
3. Incorporate a combinatoric strategy to adjust lower fidelity levels to $F_0^j(\mathbf{x})$

1. *Initial Sampling*

Constructing an aerodynamic database from scratch requires information from the entire domain of interest. To complete this objective, we are interested in implementing an efficient space filling sampling procedure. This leads us to consider Latin Hypercube Sampling. We have also incorporated a lattice design, but when combined in our methodology, resulted in samples further away from our locations of interest and additional uncertainty in parameters. Potentially more interesting than determining how to disperse a fixed number of samples over the design space is calculating where to place new points to satisfy the objective function as described in our optimization formulation. Part of our work in the future will be to incorporate where the database is being queried to analyze the certification maneuvers and the uncertainty versus cost trade to construct different optimization objectives.

2. *Surrogate Models*

Having data samples at fixed locations is great when the aircraft is at those particular flight conditions, however, when the aircraft is not, an interpolation or extrapolation method is necessary to provide the aerodynamic data. Surrogate models are a good way of mapping a sparse amount data to cover the entire design space. When using surrogate-based models to construct the aerodynamic database, we assume each fidelity level uses some subset of the m input parameters, $\mathbf{x}_i \in \mathbb{R}$, but the outputs, f_i^j for j = 1,...,p can vary with fidelity level where p(i) is the number of outputs of information source i. We are constructing our response surface using the DACE Toolbox in MATLAB.[6] For fidelity level i, the surface is constructed using:

$$F_i(\mathbf{x}) = \mathbf{b}_i(\mathbf{x})^T \beta_\mathbf{i} + \mathbb{Z}_i(\mathbf{x}) \tag{3}$$

where $\mathbf{b}_i$ and $\beta_\mathbf{i}$ are the basis functions and the coefficients, respectively, of a linear model. $\mathbb{Z}_i$ is modeled as a zero-mean stationary Gaussian stochastic process with covariance between points $\mathbf{x}$ and $\mathbf{x}'$ calculate as $\sigma^2_{\mathbb{Z},i}\mathbf{R}$. $\mathbf{R}$, we assume, is a Gaussian correlation:

$$\mathbf{R} = Corr\left[\epsilon_i(\mathbf{x}), \epsilon_i(\mathbf{x}')\right] = \prod_{j=1}^{q} exp\left[-\theta_j(x_j - x_j')^2\right] \tag{4}$$

where q is the dimension the response surface covering and θ_j is the hyper-parameter associated with a specific dimension of j. The possible aerodynamic characteristic is modeled by a normal random variable with mean μ and variance σ^2. This response surface is commonly known as a Gaussian Process (GP). An estimate for the mean of the function is

$$\hat{\mu} = \frac{\mathbf{1}^T\mathbf{R}^{-1}\mathbf{y}}{\mathbf{1}^T\mathbf{R}^{-1}\mathbf{1}} \tag{5}$$

where $\mathbf{y}$ is a vector containing the coefficient samples at the fidelity level of interest. The hyper-parameters (θ_j and $\sigma^2_{\mathbb{Z},i}$) are calculated by finding the Maximum Likelihood Estimator (MLE). $\sigma^2_{\mathbb{Z},i}$ can be calculated using,

$$\hat{\sigma}^2_{\mathbb{Z},i} = \frac{(\mathbf{y} - \mathbf{1}\hat{\mu})^T\mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu})}{n} \tag{6}$$

, with n representing the number of samples used. The output variance from the Gaussian Process at a sampled location is defined

$$s^2 = \hat{\sigma}^2_{\mathbb{Z},i}\left[1 - \mathbf{r}^T\mathbf{R}^{-1}\mathbf{r} + \frac{(1 - \mathbf{1}^T\mathbf{R}^{-1}\mathbf{r})^2}{\mathbf{1}^T\mathbf{R}^{-1}\mathbf{1}}\right] \tag{7}$$

denoting $\mathbf{r}$ as a vector containing $r_i = Corr[\epsilon(\mathbf{x}), \epsilon(\mathbf{x^i})]$ for each separate sampling location i. A separate response surface is constructed for each aerodynamic derivative necessary for the simulation at each fidelity level where that derivative can be calculated. These different response surfaces must then be connected using some multi-fidelity combinatorial strategy.

3. *Multi-Information Source Combinatorial Strategies*

To combine the multiple fidelity levels into one stochastic database we will use a additive combinatorial strategy

$$F^j_{l(j)}(\mathbf{x}) = F^j_{l(j)+1}(\mathbf{x}) + \alpha^j_{l(j)}(\mathbf{x}) \tag{8}$$

where l(j) is some subset of information sources i that provide information for quantity of interest j over the the design space modifying the work of Huang et al. (2006).[7] The difference between models at fidelity level l(j) and l(j)+1 would be constructed using a response surface $\alpha^j_{l(j)}(\mathbf{x}_l) = F^j_{l(j)}(\mathbf{x}_{l(j)}) - F^j_{l(j)+1}(\mathbf{x}_{l(j)})$ from the sample data at the fidelity levels of interest. At the lowest level, $\alpha^j_{q(j)}(\mathbf{x}_{q(j)}) = F^j_{q(j)}(\mathbf{x}_{q(j)})$ with q(j) being the lowest fidelity level that supplies output j. The multiple information sources and their respective response surfaces can then be combined to build a stochastic distribution F^j_0 $\forall$ j's where each j corresponds a different piece of information to be stored in the aerodynamic database. From this stochastic database, we can then create a deterministic sample to use in the analyses of interest.

B. Creating a Deterministic Instance of the Aerodynamic Database

When traditional aerodynamic databases are integrated into flight simulations, the aerodynamic characteristics of interest are stored at specific flight conditions (M, α, q, ...) and then interpolated between neighboring points to calculate an estimate of the characteristic at the condition of interest. Within our methodology, we decide to instead take the stochastic database and sample at the location of interest using a multivariate normal constructed from all the different characteristics stored in our database. The probability density function of a multivariate normal is

$$f_y(y_1, ..., y_k) = \frac{1}{\sqrt{(2\pi)^k|\Sigma|}} exp\Big(-\frac{1}{2}(\mathbf{y} - \mu)^T\Sigma^{-1}(\mathbf{y} - \mu)\Big) \tag{9}$$

where $\mathbf{y}$ contains the k different aerodynamic derivatives stored in the stochastic database. The μ terms are the mean of each response surface at the location of interest and the covariance matrix Σ is constructed as follows

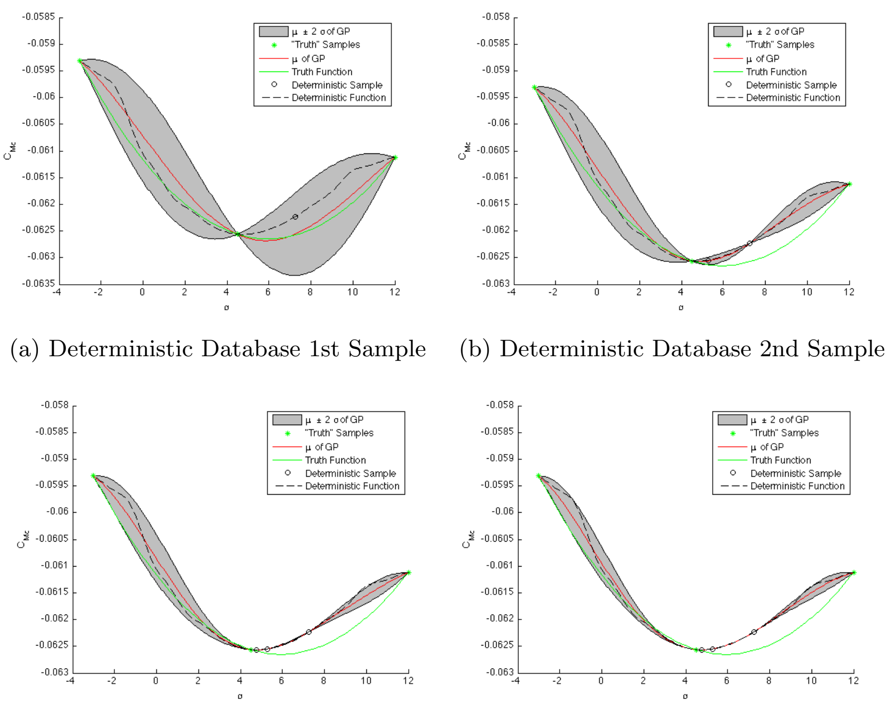
$$\Sigma_{i,j} = \rho_{i,j} s_i(\mathbf{x}) s_j(\mathbf{x}) \tag{10}$$

with $s_i(\mathbf{x})$ denoting the standard deviation of the Gaussian Process for characteristic i at $\mathbf{x}$. The correlation between parameters is calculated using the sample correlation function:

$$\rho(\mathbf{y_j}, \mathbf{y_k}) = \frac{\frac{1}{n_0-1}\sum_{i=1}^{n_0}\Big((y_{j,i}-\mu_j)(y_{k,i}-\mu_k)\Big)}{\sigma_j \sigma_k} \tag{11}$$

where μ_j denotes the mean of characteristic j over the entire aerodynamic database, σ_j denotes the estimated standard deviation, and n_0 is the number of samples seen up to that point. We calculate the correlation using the "truth" data and the points queried previously in that same deterministic instance. This methodology requires an assumption be made that the correlation between characteristics remains constant over the entire aerodynamic database. Future research will address the validity of this assumption.

Once one set of deterministic samples $\mathbf{y}^i$ of the stochastic database has been calculated from the multivariate Gaussian at the flight condition of interest, an updating procedure is followed. Coefficients at additional flight conditions are then found with the multivariate Gaussian conditioned on the deterministic instances generated at previous flight conditions sampled. The procedure is repeated, updating the conditional distribution, until deterministic instances of the stochastic database are generated for all points in the maneuvers of interest. Unlike the traditional methodology of storing deterministic values over the entire aircraft domain and interpolating to generate results, this methodology will calculate the desired aerodynamic coefficients at any sampling location ($\mathbf{x^k}$) specified using the multivariate normal of the response surface data. In one dimension, this methodology is illustrated in Figure 2.



(a) Deterministic Database 1st Sample (b) Deterministic Database 2nd Sample

(c) Deterministic Database 3rd Sample (d) Deterministic Database 4th Sample

Figure 2: Constructing a Deterministic Aerodynamic Database

It should be remembered that a multivariate normal create an infinite number of possible output instances. The shaded area in Figure 2 shows the possible data values falling within two standard deviations of the response surface mean. The black dotted line, our deterministic function, in Figure 2 is one of the possible estimates of the true underlying function, denoted as the green line, which is unknown. As more samples are seen in the maneuver, the stochastic database conditioned on these points has a better estimate of the other outputs and then uncertainty area, illustrated as the gray, shrinks. The deterministic aerodynamic database would be used to calculate the forces and moments being experienced during the maneuver. If parts of the aerodatabase are not necessary for the maneuvers, then those coefficients are not generated.

Since Gaussian Process samples condition future estimates, this methodology removes the requirement that an entire deterministic aerodynamic database be stored to run maneuvers of interest. It is flexible enough, however, where the stochastic database can be queried at a set of points for incorporation into a legacy code where a certain aerodynamic database input format is expected. Overall, we believe this methodology offers the potential to decrease the computational cost required for maneuver simulations since only locations of interest are ever specifically calculated and no interpolation is required, but we do acknowledge that updating the response surface will require additional expense. The data storage requirements for the aerodynamic database are decreased, however, since only queried locations are saved. With this methodology in place to instance the stochastic database constructed using Gaussian Processes, we have the capability to analyze aircraft through specific certification maneuvers.

V. Application of Interest

To test our methodology for a real world problem of interest, a vehicle configuration must be defined, a set of information sources must be chosen, and a maneuver of interest must be specified. For our work, we will simulate the NASA CRM using conceptual design handbook methods and QuadAir[8] , a compact Vortex Lattice Method (VLM), through an emergency descent maneuver.

A. NASA CRM

The NASA CRM configuration was developed to be used in a CFD (Computational Fluid Dynamics) validation exercise as part of the fourth AIAA (American Institute of Aeronautics and Astronatics) CFD Drag Prediction Workshop.[9] This is a high-speed configuration where the geometry is widely available and studied extensively. State-of-the-art tools were used to design the configuration with particular emphasis placed on the aerodynamic design of the wing. Throughout the design, the goal of CFD validation drives configuration decisions over minimizing drag. The resulting vehicle is a low-wing standard tube-and-wing configuration with a flight design Mach number of 0.85. The Boeing Company created the primary aerodynamic design and NASA FA (Fundamental Aerodynamics)/ SFW (Subsonic Fixed Wing) held the responsibility for model design, fabrication, and testing.

For the on-design condition at Mach 0.85, a nominal lift condition of $C_L = 0.5$ at Reynolds number of Rn = 40 million per reference chord is set. The reference quantities for the CRM main wing are located in Table 1 where λ is the taper ratio, Γ is the wing sweep, and the X, Y, Z reference locations are for half the main wing, not the entire vehicle. This final configuration stemmed from a 5-point optimization of the Mach number and C_L for wing-body model with constraints on the wing thickness and spanload distribution using OVERFLOW.[9] After the initial optimization, a nacelle/pylon component was installed to make a more realistic geometry. A horizontal tail, with characteristics shown in Table 2, designed to be robust at dive Mach number conditions was added for stability and control considerations.

This tail is integrated into the vehicle at three different incidence angles: $-2^o, 0^o$ and $+2^o$. The different models available for study include

1. wing/body
2. wing/fuselage/pylon/nacelle
3. wing/fuselage/horizontal-tail ($i_h = -2^o, 0^o, +2^o$)

where we have chosen to use the horizontal tail configuration with horizontal tail incidence angle ($i_h = 0^o$). At present, a vertical tail has not been designed for the CRM. We added a vertical tail for the CRM based on the 3-view of the Boeing 777-200[10] with estimates of the other salient characteristics shown in Table 3.

All the characteristics defined in Table 3 are estimated results and should not be considered definitive characteristics of the CRM. In addition to constructing a vertical tail for the CRM, we created control

Table 1: Reference Quantities for the CRM main wing[9]

Sref	594,720.0 in^2	4,130.0 ft^2
Cref	275.8 in	
Span	2,313,5 in	192.8 ft
Xref	1,325.9 in	
Yref	468.75 in	
Zref	177.95 in	
λ	0.275	
$\Lambda_{C/4}$	35^o	
AR	9.0	

Table 2: Reference Quantities for the CRM Horizontal Tail

Sref	144,000.0 in^2	1,000.0 ft^2
Cref	184.7 in	
Span	840 in	70 ft
λ	0.35	
$\Lambda_{C/4}$	37^o	

Table 3: Salient Characteristics for an Estimated CRM Vertical Tail

Sref	86,141.1 in^2	598.2 ft^2
Cref	239.5 in	19.95 ft
Span	393 in	32.75 ft
λ	0.31	
$\Lambda_{C/4}$	45^o	
t/c_{root}	0.1	

surfaces for the vehicle since we are trying to simulate a vehicle through certification maneuvers. At present, we are only considering logitudinal maneuvers. With this in mind, we incorporate an elevator and set of spoilers base on the Boeing 777-200[10] to control the CRM through the desired trajectories. The control surface assumed geometric quantities are in Table 4.

Table 4: Assumed Geometric Specifications for the CRM Longitudinal Control Surfaces

$c_{\delta e}/c_h$	0.2849
$Span_{\delta e}$	524.7 in^2
n_{spoil}	5
$Span_{spoil}$	84.12 in
$Cref_{spoil}$	21.05 in

In our construction of the CRM, similar to the Boeing 777-200, each of the five spoilers is rectangular with the defined geometric characteristics defined above. The aerodynamic effects of the spoilers will be defined in their own analysis component, separate from the aircraft analysis, described below. A picture of the CRM with horizontal tail, but without vertical tail and control surfaces is shown in Figure 3.

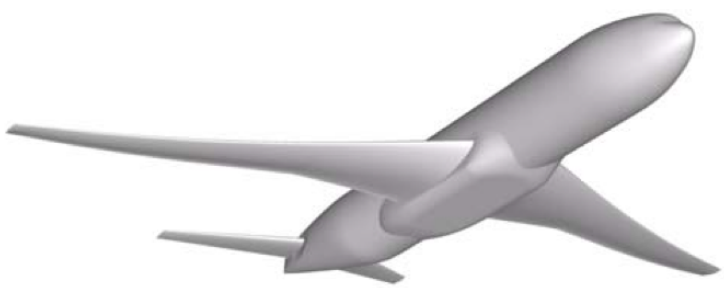


Figure 3: CRM Wing-Body-Tail Configuration[11]

The CRM does not have a defined mass or moment of inertia. As these parameters are necessary for our analysis, we will assume the CRM mass is 545,000 lbs, the same as the Boeing 777-200[10] , and the moment of inertia I_{yy} is estimated at $2.4e7\ slugs - ft^2$. We also assume the CRM has two 77,000 lbs thrust engines in the same locations as the Boeing 777-200.

B. Information Sources

When determining what information sources to include in our analysis, the problem specific goals and computer budget available must be considered. If the user is a conceptual design engineer who wants to analyze a new configuration every week, day, or as part of an integrated optimization, then using tools such as LES or multi-million cell RANS to populate the aerodatabase is probably not feasible. However, if a design is further along and the engineer wants to gain an understanding of how the vehicle might perform in a wind tunnel or potentially a flight test, the higher fidelity sources become more computationally traceable. Since our goal at present is to develop a methodology for sampling any number of information sources, we desire a set of rapid generating information sources so less time is spent generating aerodynamic data and more time devoted to understanding, improving, and correcting the construction and updating of the aerodynamic database. With that goal in mind, we chose a two level tool hierarchy where the low fidelity is a compilation of handbook methods and the high fidelity analysis is a compact VLM. To make sure that we are generating tools that are at least giving reasonable results for the aerodynamic analysis, even though we realize that tools could be much more accurate, we will compare these tools to available wind tunnel data for the CRM.

1. *Handbook Methods*

For our low fidelity aerodynamic analysis, we have constructed a collection of handbook methods from various sources to generate all the aerodynamic rate coefficients of interest. $C_{L_{\alpha_i}}$ is calculated from the DATCOM formula[12] for each of the lifting surfaces with the aircraft C_{L_α} calculated from

$$C_{L_\alpha} = C_{L_{\alpha_w}} - \frac{S_h}{S_w} C_{L_{\alpha_h}} \tag{12}$$

The drag coefficient of zero lift, C_{D_o}, is generate from a build-up of the parasite drag combined with a compressibility drag increment.[12] C_{M_α} is calculated from

$$C_{M_\alpha} = \left(\frac{C_M}{C_L}\right)_w C_{L_{\alpha_w}} - \frac{S_h}{S_w}\left(\frac{C_M}{C_L}\right)_h C_{L_{\alpha_h}} \tag{13}$$

with each $\left(\frac{C_M}{C_L}\right)_i$ determined by

$$\left(\frac{C_M}{C_L}\right)_i = \frac{n - 2/3(1-\lambda) + 0.5(1 - \frac{\lambda^2}{1+\lambda}\pi ln(1+AR/5)}{1+\pi ln(1+AR/5))}\frac{c_{root}}{\bar{c}} \tag{14}$$

where n is the distance from wing apex to desired moment reference center measured in root chords. C_{M_o} incorporates thin airfoil theory analysis of c_{m_o} (the airfoil zero pitching moment) into

$$C_{M_o} = -\frac{AR_w cos^2(\Lambda_w)}{AR_w + 2cos(\Lambda_w)} c_{m_o} \tag{15}$$

C_{L_o} is set to zero in our analysis by comparing with reference data. $C_{M_{\delta e}}$ uses the DATCOM[13] estimate of the section lift-effectiveness parameter ($a_{\delta e}$) correct by the formula

$$C_{M_{\delta e}} = C_{L_{\alpha_h}}\left(\frac{C_M}{C_L}\right)_h a_{\delta e} \tag{16}$$

The pitch rate terms (C_{L_q} and C_{M_q}) are calculated following the analysis of Roskam.[14] The handbook aerodynamic coefficients are compared to reference data and other information sources in Figure 5. The coefficients have been generated from the coefficient rate of changes defined above following the methodology of Stevens and Lewis.[4] Since the CRM has no control surfaces and has no rate wind tunnel data available, only a subset of the coefficients created can be compared to actual data. These equations are representative of a zeroth order analysis available to a design engineer and act as a rapid tool to be corrected by the higher fidelity information sources using our methodology. We assume in our present work that the handbook method is cheap enough that it can be called at any aerodynamic database point of interest. This reduces the number of necessary response surfaces and thus the computational expense in running each simulation without limiting our methodology. If lowest level analysis is still too expensive for every data point of interest, then a surrogate model can be fit to the lowest fidelity data to create a function to span the entire flight domain.

2. *QuadAir*

As a high fidelity information source, we have decided to use QuadAir, the compact VLM method developed by Bunge and Kroo.[8] This formulation captures the nonlinearities of the standard inviscid VLM without a costly re-computation step. We thus can quickly solve the equations of interest to generate the necessary results. The code is also open-source and can be easily modified to generate aerodynamic derivatives for the aerodatabase instead of the coefficients normally output from other vortex lattice methods. A picture of the CRM model analyzed in QuadAir is shown in Figure 4, where only the lifting surfaces are modeled.

The aerodynamic coefficients of interest in the longitudinal maneuvers are shown with data from the other information sources in Figure 5. As a basic aerodynamic analysis tool used to demonstrate our methodology, QuadAir exhibits highly desirable characteristics, such as speed of analysis, close approximation of the aerodynamic quantities of interest, and the ability to inspect the underlying code to understand the input-output relationship. As QuadAir is our so-called "truth", we will calculate the sample correlation between its output terms to build an estimated correlation matrix for the stochastic database. One possible

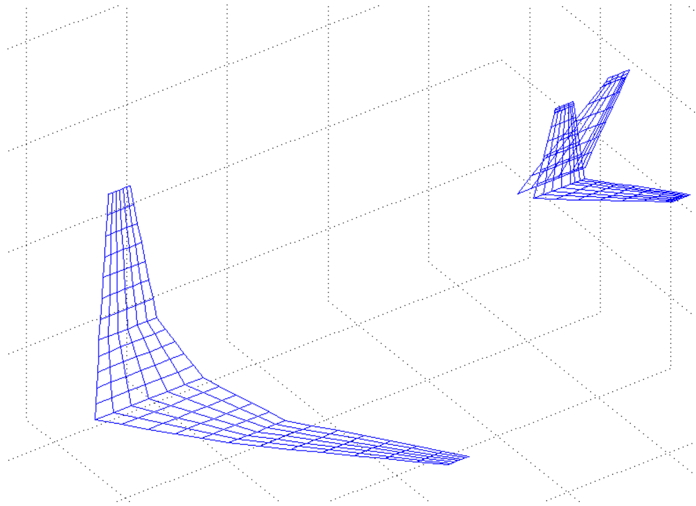


Figure 4: QuadAir model for the CRM

Table 5: QuadAir Correlation Matrix for 200 LHS Points

$$
\begin{array}{cccccccc}
C_{L_\alpha} & C_{M_\alpha} & C_{D_o} & C_{M_o} & C_{M_{\delta e}} & C_{L_o} & C_{L_q} & C_{M_q}
\end{array}
$$

$$
\begin{bmatrix}
1 & -0.2155 & 0.7765 & -0.0129 & 0.0390 & -0.0082 & 0.5613 & -0.5451 \\
-.2155 & 1 & 0.1556 & -0.0055 & -0.4121 & 0.0007 & -0.2068 & 0.2204 \\
0.7765 & 0.1556 & 1 & -0.0116 & 0.0337 & -0.0061 & 0.4823 & -0.4642 \\
-0.0129 & -0.0055 & -0.0116 & 1 & 0.0166 & -0.9764 & -0.0089 & 0.0088 \\
0.0390 & -0.4121 & 0.0337 & 0.0166 & 1 & 0.0090 & -0.0358 & 0.0280 \\
-0.0082 & 0.0007 & -0.0061 & -0.9764 & 0.0090 & 1 & -0.0078 & 0.0073 \\
0.5613 & -0.2068 & 0.4823 & -0.0089 & -0.0358 & -0.0078 & 1 & -0.9989 \\
-0.5451 & 0.2204 & -0.4642 & 0.0088 & 0.0280 & 0.0073 & -0.9989 & 1
\end{bmatrix}
$$

correlation matrix between the eight aerodynamic characteristics of interest constructed from a 200-point Latin Hypercube Sampling of QuadAir is shown in Table 5.

Comparing the correlation between C_{L_α} and C_{M_α}, we see as expected there is a negative correlation between terms. In addition, looking at the correlation between C_{M_o} and C_{D_o}, not surprisingly there is little underlying correlation. As discussed in the stochastic aerodynamic database section, this correlation matrix is combined with the sample standard deviation calculation from the Gaussian Processes to construct the covariance matrix Σ necessary to generate the multivariate normal distribution of the output samples. The covariance matrix is calculated at the highest fidelity level, QuadAir in our analysis, but could easily be instead calculate by some higher-fidelity simulation, LES, or experimental data obtained from flight or wind tunnel tests.

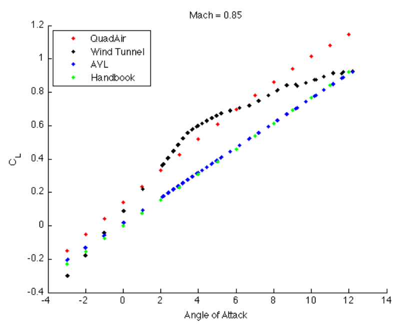
3. NASA Wind Tunnel

As part of the analysis of the CRM for the Drag Prediction Workshop, a set of wind tunnel tests were conducted at the NASA Langley National Transonic Facility and at NASA Ames in the 11-ft wind tunnel.[15] The five different vehicle configuration mentioned above were tested in the facility at a Reynolds Number of 5 million. We have shown the results for the Wing/Fuselage/Horizontal Tail ($i_h = 0$)

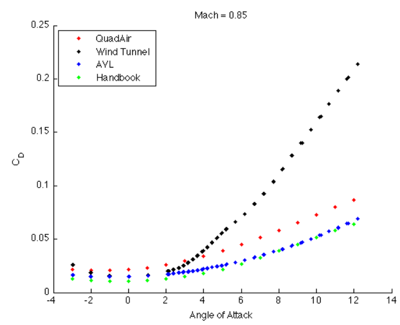
A correction for the C_{M_α} for both QuadAir and the handbook methods to account for the fuselage effect[12] is

$$C_{M_\alpha} = C_{M_\alpha} + \frac{K_f d_{fuse}^2 l_{fuse}}{C_{L_{\alpha_w}} S_{wing} \bar{c}_{wing}} C_{L_\alpha} \tag{17}$$

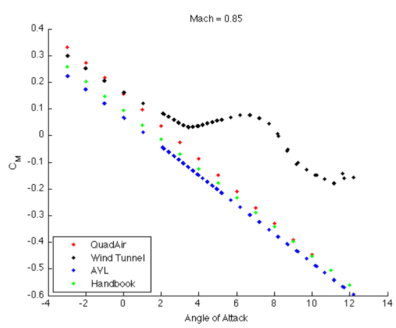
where $\bar{c}$ is the Cref defined above in Table 1 and K_f is a correction constant equal to 0.688 for our problem. These corrected results for each of the information sources are shown in comparison to the wind tunnel data in Figure 5.



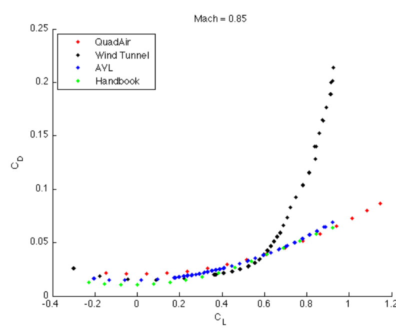
(a) Lift Coefficient



(b) Drag Coefficient



(c) Pitching Moment Coefficient



(d) Drag Polar

Figure 5: Comparison of Different Information Sources with the CRM Wind Tunnel Data

Comparing the methods with wind tunnel data, we see our lower fidelity results have significant error at large angles of attack. We, however, are primarily interested in angles of attack between 2 and 4 degrees. In this range, our tools do a reasonable job of predicting the aerodynamic coefficients.

AVL[16] is also shown in this comparison as QuadAir results should closely follow AVL since both are vortex lattice codes. The differences between AVL and QuadAir stem from the discrepancy in models. Comparing the AVL in model in Figure 6 and the QuadAir model shown above in Figure 4, the AVL model does not corporate the vertical tail at present, but does add the fuselage, which the AVL model lacks. Since we are focusing on longitudinal maneuvers, this difference should not significant. Overall, at this stage of the analysis, the two information sources we are combining should do a reasonable job of generating aerodynamic data rapidly for testing our mathematical formulation.

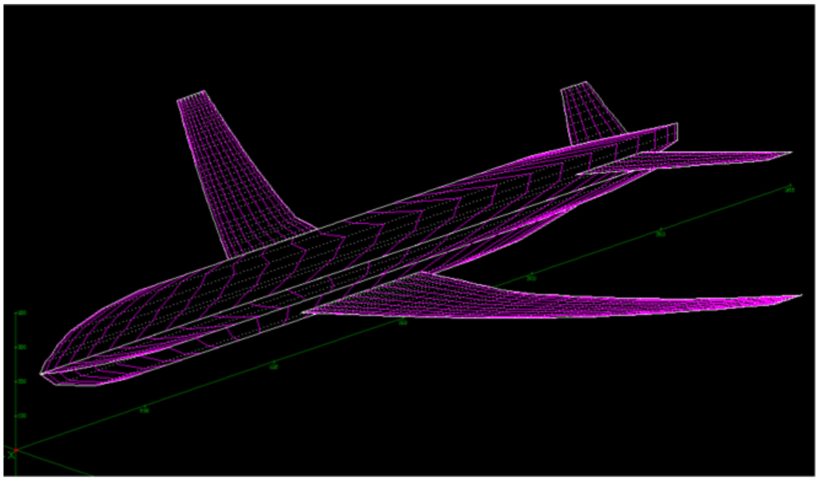


Figure 6: AVL Model of the CRM

C. Spoiler Analysis

As part of our work, we are interested in determining how spoilers affect the vehicle. Spoilers analysis is an area of continued research. Their effect is dominated by flow separation behind the deflected control surface. We thus decided to incorporate quick handbook analysis techniques at present with the flexibility to add higher fidelity tools as our analysis deems necessary. With that in mind, we look to correct C_{L_o} and C_{M_o} from the stochastic aerodynamic database using a model for miniature trailing edge effectors by Lee[17]

$$\Delta C_{L_o} = 2(\theta_s + sin\theta_s)\delta_s \tag{18}$$

$$\Delta C_{m_o} = -\frac{1}{2} sin\theta_s(1 + cos\theta_s)\delta_s \tag{19}$$

where δ_s is the spoiler deflection and θ_s is calculated from

$$\frac{c_s}{c} = \frac{1}{2}(1 - cos\theta_s) \tag{20}$$

with c being the reference chord at the wing station of the spoiler. C_{D_o} corrected following the work of Sadraey[18]

$$\Delta C_{D_o} = 1.9 sin\delta_s \frac{S_s}{S_w} \tag{21}$$

Once all the aerodynamic effects have been calculate with respect to spoiler deflection, determining the spoiler deflection is necessary. When in normal flight, aircraft use spoilers as a secondary control device, such as roll control at high speeds. However, there are certain times where the maximum deflection of the spoilers might be required. At these instances, the deflection is governed by the maximum torque the actuator can supply. The maximum torque will equal the maximum blow-back force generated by the airflow over the aircraft. Assuming a triangular pressure profile over the spoiler following the work of Stoecklin,[19] the torque of the spoiler is

$$\tau_s = \frac{b_s c_s^2 S_w \bar{q} \sqrt{\Delta C_{D_{\delta_s}}^2 + \Delta C_{L_{\delta_s}}^2}\, \delta_s}{3 S_s} \tag{22}$$

During every flight evaluation state of interest, an unconstrained minimization of the absolute value of the difference between τ_s and τ_{max} is completed for each spoiler to determine the maximum deflection with a maximum spoiler deflection, $\delta_{s_{max}}$, set to 60 degrees. This flexibility will allow us to study many different maneuvers that might have different spoil requirements.

D. Certification Maneuver

When considering different limiting certification maneuvers of interest for aircraft designs, one that repeatedly comes up is the emergency descent maneuver. As written into FAR 25.841 related to cabin pressures, if an aircraft wants to be certified above 25,000 ft, then "the airplane must be designed so occupants will not be exposed to cabin pressure altitude that exceeds the following after decompression from any failure condition not shown to be extremely improbable:

1. Twenty-five thousand (25,000) feet for more than 2 minutes; or

2. Forty thousand (40,000) feet for any duration."[20]

This requirement is one that helps guide the aircraft designers to determine the maximum cruise altitude of the aircraft. In addition, the maneuver can be solved using only longitudinal equations of motion simplifying the analysis. We will make the assumption that cabin pressure equals external pressure. The aircraft will then meet the requirement if aircraft altitude is above 25,000 ft for less than 2 minutes. With these assumptions, this analysis seems to be an ideal case study for our stochastic simulation methodology.

When this maneuver is conducted in general, there are three major segments between which the aircraft transitions. The aircraft starts in the trim cruise condition when a warning alerts the pilots to a cabin depressurization. At that time, the pilots pitch the aircraft over and trims the vehicle to descend at M_{MO}, the maximum operating Mach number of the aircraft. During the descent profile, the aircraft reaches a transition altitude where the aircraft must retrim at V_{MO}, the maximum operating velocity. The vehicle continues to descend at this condition until the flight altitude decreases below 25,000 ft when the maneuver time is stopped. The aircraft, however, continues to descend down to a lower cruise altitude of 10,000 or maybe 15,000 ft. Since the maneuver after the aircraft altitude below 25,000 ft is not part of the certification requirement, we will not be concerned with this portion of the trajectory in our test. As a way to avoid doing a complete non-linear simulation of the maneuver, which can be quite expensive, we instead are using control point analysis where we discretize the descent profile into a fixed set of locations defined by an altitude and Mach number. The aircraft must then be trimmed at each of these locations by querying the stochastic database.

1. *Trimming the Aircraft*

To trim the aircraft, we are using the set of equations based on the work of Stevens and Lewis.[4] An inner sizing loop is used to match the spoiler torque to the maximum specified torque for each of the spoilers as discussed in the Spoiler Analysis section above. The three total vehicle requirements forced to zero for the aircraft to be considered trim are rate of change of vehicle velocity ($\dot{V}_T$), rate of change of angle of attack ($\dot{\alpha}$), and the rate of change of the pitch angle rate of change ($\dot{q}$). These three objectives are combined using

$$f = \dot{V}_T^2 + 100\dot{\alpha}^2 + 10\dot{q}^2 \tag{23}$$

Newton's method is used to converge the aircraft simulation incorporating analytic gradients where the deterministic aerodatabase is queried at each iteration. The inputs for this optimization are the elevator deflection angle (δe), the aircraft angle of attack (α), and the flight path angle (γ).

2. Quantity of Interest

For an emergency descent maneuver, the simulation output of interest is the time to descend. To calculate the transit time for each control point ($t_{D_{s_j}}$), we use

$$t_{D_{s_j}} = \frac{\Delta h_j}{V_{T_j} sin(\gamma_j)} \tag{24}$$

where Δh_j is the altitude to descend at the trim condition specified, V_{T_j} is the trim flight velocity at the control point, and γ_j is the trimmed aircraft flight path angle. We assume the control points are located in the center of the maneuver section. The total maneuver time (t_D) is the sum of the individual segment descent times

$$t_D = \sum_{j=1}^{N_M+N_V} t_{D_{s_j}} \tag{25}$$

denoting the number of control points in the M_{MO} flight phase by N_M and the number of control points in V_{MO} by N_V.

These descent times are combined together using a UQ methodology (Monte Carlo in our results) to generate statistics of interest (mean, variance, 95% confidence interval, etc.). These statistics can then be incorporated into the optimization constraints and the QoI. Distribution profiles can also be generated and will be shown in Results.

VI. Results

To gain insight into what is an appropriate objective function to use when conducting the optimization defined in Section II, we will simulate the CRM through an emergency descent maneuver. We are also interested in understanding how the number of trim control points affects the results for time to descend. Figure 8 shows how Figure 1 would be formulated in this case of interest.

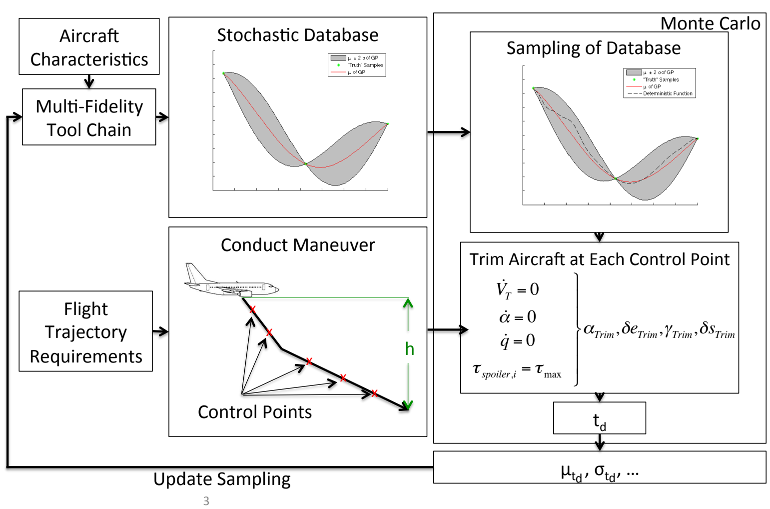
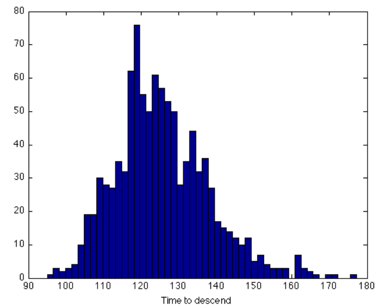


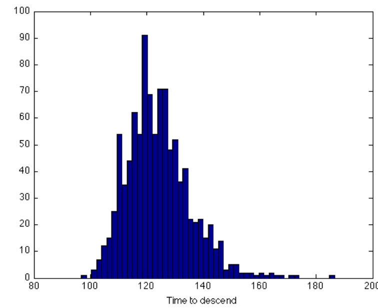
Figure 7: Diagram of Problem for CRM Descent Maneuver

A. Descent Time Distribution

Using the analysis capabilities described in Section V, we can generate output distributions of the time to descend. We define the torque for each spoiler to be $\tau_{max} = 5,500 ft - lbs$ and the idle throttle setting to be 0.15. In addition, the stochastic database is constructed for Mach number, α, and δe. Mach number range of interest is 0.8-0.92, α range is -3 to 12 degrees, and δe range is -25 to 25 degrees. These ranges were estimated based on the cruise conditions of 777 and with an approximate increase in Mach number due to emergency dive condition. The stochastic database was initially sampled at 15 points using a LHS approach. V_{MO} was set to 900 ft/s and M_{MO} is 1.08 times the cruise Mach number, 0.84, or 0.9072. Computing the descent maneuver with 1,000 different instances of the stochastic database, the output distribution is shown in Figure 8a using one control point to approximate the M_{MO} descent and one control point for V_{MO} descent and Figure 8b shows 16 points for both M_{MO} and V_{MO} descent. The initial cruise altitude was 35,000 ft.



(a) 1 point each for V_{MO} and M_{MO}



(b) 16 points each for V_{MO} and M_{MO}

Figure 8: Distribution of CRM Emergency Descent Time

From the 1 control point analysis, the mean time to descend was 125.50 seconds and the standard deviation was 12.4 seconds. The 16 control point analysis had mean 124.14 seconds with standard deviation 11.3 seconds. From these results, we see that there is not a very large discrepancy in the number of control points used.

If we decrease the number of LHS samples to 10 for the initial sampling and use 1 trim state for V_{MO} descent and 2 for M_{MO} descent, but sample the database at locations determined by different objective functions to see how this affects the output distribution. Possible objectives to use for optimization include maximum product of all uncertain terms at one flight condition, maximum uncertainty for the dominate aerodynamic characteristics, C_{D_o} in our analysis, or maximum uncertainty of any analysis characteristic (denoted σ_j^2). What we are really interested in finding is the point in our stochastic database that corresponds with the maximum uncertainty in the time to descend distribution. At current time, we do not have a closed form solution to this maximum uncertainty parameter, but if we sample at one of our trim points, we can identity the impact of sampling at an improved point instead of just the most uncertain location. The results in Table 6 show how these sampling strategies affect the output distribution characteristics. When we have multiple certification maneuvers from one database, the objective function will have to be reformulated to incorporate the uncertainty resulting in using one aerodatabase for both maneuvers.

Table 6: Comparing Updated Sampling Locations with Change in Output Distribution

Objective	M_{Samp}	α	δe	Mean	Standard Deviation
Initial Sampling	-	-	-	133.73	22.35
Product σ^2	0.80	0	-24.99	128.57	20.71
C_{D_o} σ^2	0.8345	6.23	24.95	130.08	19.71
σ_j^2	0.8	-2.85	24.57	130.15	21.51
$\sigma^2(t_d)$	0.9072	2.1833	0.4783	129.11	5.02

B. Comparison to current analysis capabilities

This analysis gives the aircraft design engineer significantly more information compared to present analysis techniques. From the 1 control point for each section of maneuver, the mean is 125.50 seconds. Without knowing the uncertainty in the result, it would be said the aircraft does not meet the requirements. Instead of redesigning the aircraft and potentially trading performance for a buffer in descent time, additional data points can be used to gain a better understanding of vehicle maneuver characteristics. If the vehicle will actually meet the requirement, a redesign is avoided. If the redesign is still necessary, then the designer has gained information on what characteristics are dominating the aircraft not meeting certification requirements. While we are not doing this at present, we can also incorporate distributions for the mass, moment of inertia, and other aircraft attributes to investigate their impact on the aircraft meeting all criteria. This methodology can be incorporated by conceptual aircraft designers as another analysis tool to provide information that is not currently available.

VII. Summary & Conclusions

We have developed a mathematical formulation throughout this paper to analyze certification maneuvers during the conceptual design phase. This required the building of a stochastic aerodynamic database using surrogate models incorporating multiple information sources and fidelity levels of analysis. We tested this methodology on the NASA CRM configuration using QuadAir, a compact VLM, along with a set of handbook methods prevalent throughout conceptual design. Monte Carlo sampling the Stochastic Database using a conditional distribution updating strategy based on analysis locations of interest, we generated output distributions of the probability of meeting emergency time to descend requirements. The distribution statistics provide additional data to the design engineer not previously available. Our analysis also shows that sampling at strategically important locations can have a dramatic impact on the resulting output distributions.

Our future work will extend this sampling methodology into a formalized objective to be used in optimization. We would like to incorporate higher fidelity level analysis tools, such as CFD and potentially wind tunnel data, into our framework to better represent all the information sources available. Considering multiple certification maneuvers simultaneously and formulating constraints for our optimization are other areas of research. In addition, certain UQ methodologies can be used instead of the current Monte Carlo sampling approach to speed up the time it takes to generate the output distributions. Overall, we believe that we have constructed a new methodology with the potential to push stability and control analysis further forward in the design process and manage the uncertainty in meeting certification requirements.

Acknowledgments

The authors would like to thank The Boeing Company for funding this research under grant number 115393 [IC2014-0547]. Also, Brendan Tracey for his discussions on the topic.

References

[1] Norris, G., "787-9 Test Lessons to Aid 787-10 and 777X," aviationweek.com/commercial-aviation/787-9-test-lessons-aid-787-10-and-777x, October 2014.

[2] Clifton, J. D., Ratcliff, C. J., Bodkin, D. J., and Dean, J. P., "Determining the Stability and Control Characteristics of High- Performance Maneuvering Aircraft Using High-Resolution CFD Simulation with and without Moving Control Surfaces," *AIAA ASM*, , No. 2013-0972, January 2013.

[3] Forsythe, J. F., Strang, W. Z., and Squires, K. D., "Six Degreee of Freedom Computation of the F-15E Entering a Spin," *AIAA*, , No. 2006-0901, 2006.

[4] Stevens, B. L. and Lewis, F. L., *Aircraft Control and Simulation*, John Wiley & Sons, Inc., 2nd ed., 2003.

[5] Eldred, M. S. and Burkardt, J., "Comparison of Non-Intrusive Polynomial Chaos and Stochastic Collocation Methods for Uncertainty Quantification," *AIAA Aerospace Sciences Meeting*, , No. 2009-976, January 2009.

[6] Lophaven, S., Nielsen, H., and Søndergaard, J., "Aspects of the Matlab Toolbox DACE," Tech. Rep. 2002-13, Informatics and Mathematical Modelling.

[7] Huang, D., Allen, T. T., Notz, W. I., and Miller, R. A., "Sequential Kriging Optimization Using Multiple Fidelity Evaluations," *Structural and Multidisciplinary Optimization*, Vol. 32, No. 5, November 2006, pp. 369–382.

[8] Bunge, R. A. and Kroo, I. M., "Compact Formulation of Nonlinear Inviscid Aerodynamics for Fixed-Wing Aircraft," *AIAA Applied Aerodynamics Conference*, , No. 2012-2771, June 2012.

[9] Vassberg, J. C., DeHaan, M. A., Rivers, S. M., and Wahls, R. A., "Development of a Common Research Model for Applied CFD Validation Studies," *AIAA*, , No. 2008-6919, 2008.

[10] "777-200/300 Characteristics for Airport Planning," Tech. Rep. D6-58329, Boeing Commercial Airplanes, July 1998.

[11] Sclafani, A. J., DeHaan, M. A., Vassberg, J. C., Rumsey, C. L., and Pulliam, T. H., "Drag Prediction for the NASA CRM Wing-Body-Tail Using CFL3D and OVERFLOW on an Overset Mesh," *AIAA Applied Aerodynamics Conference*, , No. 2010-4219, 2010.

[12] I. Kroo, R. S., "Aircraft Design, Synthesis and Analysis,," .

[13]Finck, R. D., "USAF Stability and Control DATCOM," Tech. Rep. AFWAL-TR-83-3048, Air Force Wright Aeronautical Laboratories, April 1978.

[14]Roskam, J., *Airplane Design: Part VI, Preliminary Calculation of Aerodynamic, Thrust and Power Characteristics*, Design, Analysis and Research Corporation, Lawrence, Kansas, 2008.

[15]Rivers, M. B. and Dittberner, A., "Experimental Investigations of the NASA Common Research Model in the NASA Langley National Transonic Facility and NASA Ames 11-Ft Transonic Wind Tunnel (Invited)," *AIAA Aerospace Sciences Meeting*, , No. 2011-1126, January 2011.

[16]Drela, M. and Youngren, H., "AVL," http://web.mit.edu/drela/Public/web/avl/, February 2014.

[17]Lee, H.-T., *Computational Investigation of Miniature Trailing Edge Effectors*, Ph.D. thesis, Stanford University, December 2006.

[18]Sadraey, M., "Spoiler Design," Daniel Webster College.

[19]Stoecklin, R. L., "Development, Manufacturing, and Test of Graphite-Epoxy Composite Spoilers for Flight Service on 737 Transport Aircraft," Tech. Rep. 132682, NASA, 1976.

[20]Federal Aviation Administration, *Section 25.841 - Pressurized Cabin*, January 1999.